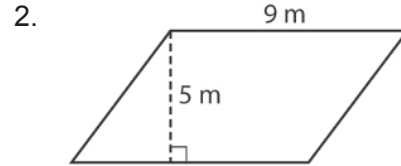
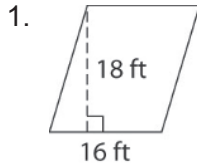


LESSON
13-1

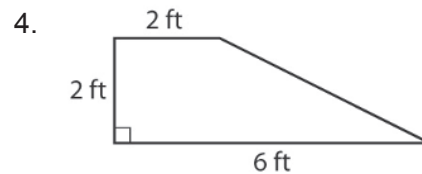
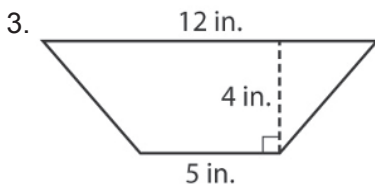
Area of Quadrilaterals

Practice and Problem Solving: A/B

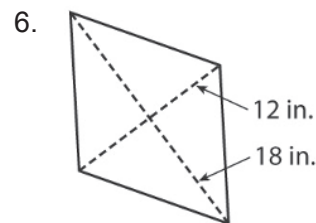
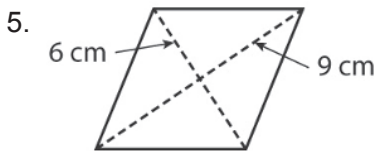
Find the area of each parallelogram.



Find the area of each trapezoid.



Find the area of each rhombus.



Solve.

7. A desktop in the shape of a parallelogram has a base of 30 inches and a height of 40 inches. What is the area of the desktop?
- _____

8. A rhombus has one diagonal that is 14 centimeters long and one diagonal that is 12 centimeters long. What is the area of the rhombus?
- _____

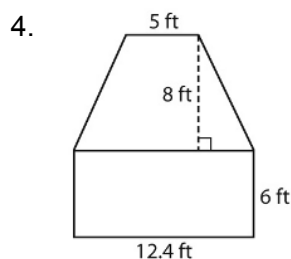
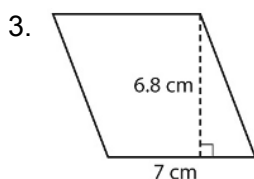
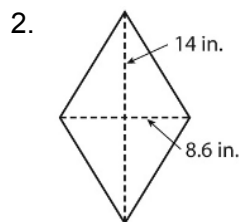
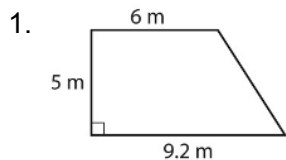
9. The bases of a trapezoid are 24 feet and 16 feet. The height of the trapezoid is 12 feet. What is the area of the trapezoid?
- _____

LESSON
13-1

Area of Quadrilaterals

Practice and Problem Solving: C

Find the area of each figure.



Solve.

5. A section of a stained-glass window is shaped like a parallelogram. Its base is 6.5 inches, and its height is 4 inches. How much glass is needed to cover the section completely?
- _____

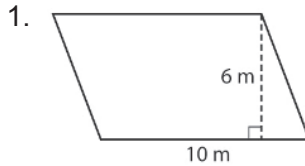
6. The base of a statue is in the shape of a trapezoid. The bases of the trapezoid are 7.5 feet and 4.75 feet. Its height is 6 feet. What is the area of the base of the statue?
- _____

7. The front view of a piece of art is in the shape of a rhombus. The front view of the art has diagonals that are 1.4 yards long and 0.8 yard long. What is the area of the front view of the piece of art?
- _____

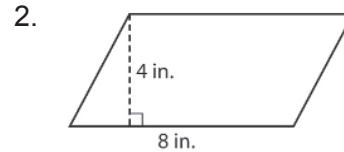
8. A decorative pillow is in the shape of a parallelogram. Its base is 28 centimeters, and its height is 24.5 centimeters. What is the area of the front surface of the pillow?
- _____

LESSON
13-1**Area of Quadrilaterals****Practice and Problem Solving: D**

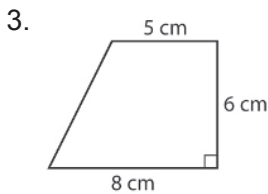
Find the area of each parallelogram. The first one is done for you.



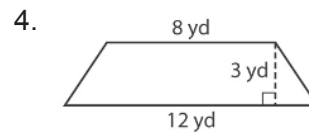
$$A = 10 \cdot 6 = 60 \text{ m}^2$$



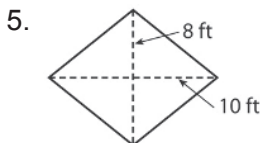
Find the area of each trapezoid. The first one is done for you.



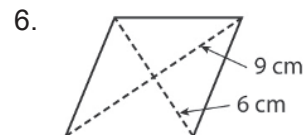
$$A = \frac{1}{2} \cdot 6 \cdot (8 + 5) = 39 \text{ cm}^2$$



Find the area of each rhombus. The first one is done for you.



$$A = \frac{1}{2} \cdot 8 \cdot 10 = 40 \text{ ft}^2$$



Solve.

7. A countertop in the shape of a parallelogram has a base of 90 centimeters and a height of 50 centimeters. What is the area of the countertop?
- _____
8. A rhombus has one diagonal that is 10 inches long and one diagonal that is 15 inches long. What is the area of the rhombus?
- _____
9. The bases of a trapezoid are 4 yards and 6 yards. The height of the trapezoid is 5 yards. What is the area of the trapezoid?
- _____

LESSON
13-1

Area of Quadrilaterals

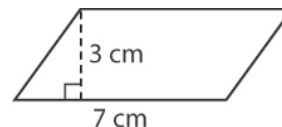
Reteach

You can use formulas to find the areas of quadrilaterals.

The area A of a **parallelogram** is the product of its base b and its height h .

$$A = bh$$

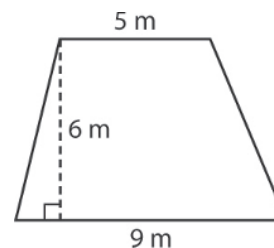
$$\begin{aligned} A &= bh \\ &= 3 \cdot 7 \\ &= 21 \text{ cm}^2 \end{aligned}$$



The area of a **trapezoid** is half its height multiplied by the sum of the lengths of its two bases.

$$A = \frac{1}{2}h(b_1 + b_2)$$

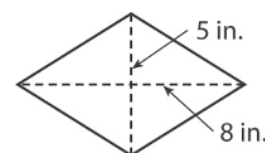
$$\begin{aligned} A &= \frac{1}{2}h(b_1 + b_2) \\ &= \frac{1}{2} \cdot 6(5 + 9) \\ &= \frac{1}{2} \cdot 6(14) \\ &= 3 \cdot 14 \\ &= 42 \text{ m}^2 \end{aligned}$$



The area of a **rhombus** is half of the product of its two diagonals.

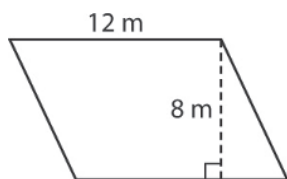
$$A = \frac{1}{2}d_1d_2$$

$$\begin{aligned} A &= \frac{1}{2}d_1d_2 \\ &= \frac{1}{2}(5)(8) \\ &= 20 \text{ in}^2 \end{aligned}$$

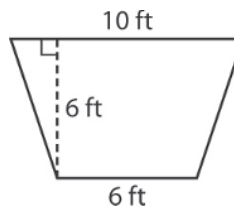


Find the area of each figure.

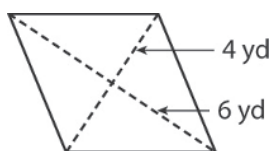
1.



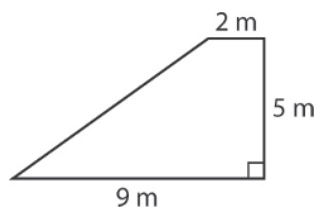
2.



3.

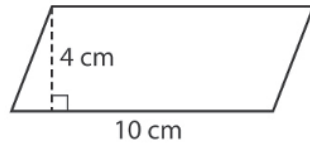
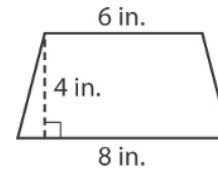


4.



LESSON
13-1**Area of Quadrilaterals****Reading Strategies: Follow a Procedure**

Parallelograms and trapezoids are two different types of quadrilaterals. You can follow a procedure to help you find the area of each type of quadrilateral.

**Step 1:** Use the formula $A = bh$.**Step 2:** Substitute the length of the base for b .**Step 3:** Substitute the length of the height for h .**Step 4:** Multiply.**Step 1:** Use the formula $A = \frac{1}{2}h(b_1 + b_2)$.**Step 2:** Substitute the length of the height for h .**Step 3:** Substitute the length of the bases for b_1 and b_2 and add.**Step 4:** Multiply.**Solve.**

1. What is the first step in finding the area of the parallelogram above?

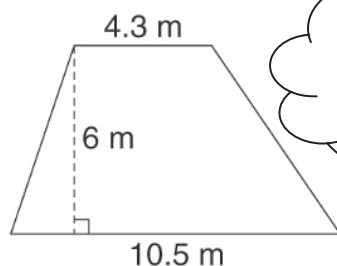
2. What are the second and third steps in finding the area of the parallelogram above?

3. What is the area of the parallelogram above?

4. What is the first step in finding the area of the trapezoid?

5. What are the second and third steps in finding the area of the trapezoid above?

6. What is the area of the trapezoid above?

LESSON
13-1**Area of Quadrilaterals****Success for English Learners****Problem 1****What is the area?****Think:**
What is the formula?

$$A = \frac{1}{2}h(b_1 + b_2)$$

Remember**A** = Area**h** = height**b₁** = base on top**b₂** = base on bottomWhat is the height? $h = 6$ mWhat is the base on top? $b_1 = 4.3$ mWhat is the base on the bottom? $b_2 = 10.5$ m

$$A = \frac{1}{2}h(b_1 + b_2) \quad \text{Use formula.}$$

$$A = \frac{1}{2}(6)(4.3 + 10.5) \quad \text{Substitute.}$$

$$A = (3)(4.3 + 10.5) \quad \text{Multiply.}$$

$$A = (3)(14.8) \quad \text{Add.}$$

$$A = 3 \cdot 14.8 = 44.4 \quad \text{Multiply.}$$

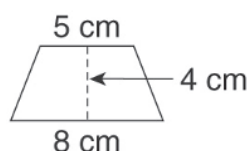
$$A = 44.4 \text{ m}^2$$

So, the area is 44.4 m^2 .

1. Does it matter which length you use for b_1 and which you use for b_2 ?
Explain.

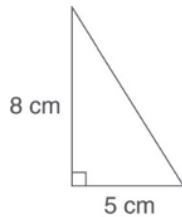
2. Describe another way to find the area of a trapezoid.

3. Find the area of the trapezoid shown.

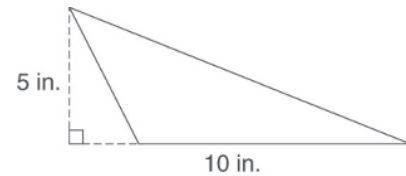


LESSON
13-2**Area of Triangles****Practice and Problem Solving: A/B****Find the area of each triangle.**

1.



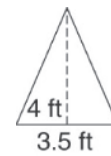
2.



3.



4.

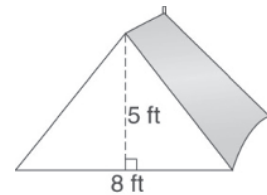
**Solve.**

5. The front part of a tent is 8 feet long and 5 feet tall. What is the area of the front part of the tent?

6. Kathy is playing a board game. The game pieces are each in the shape of a triangle. Each triangle has a base of 1.5 inches and a height of 2 inches. What is the area of a game piece?

7. A triangular-shaped window has a base of 3 feet and a height of 4 feet. What is the area of the window?

8. Landon has a triangular piece of paper. The base of the paper is $6\frac{1}{2}$ inches. The height of the paper is 8 inches. What is the area of the piece of paper?

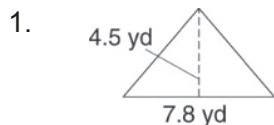


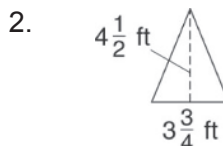
LESSON
13-2

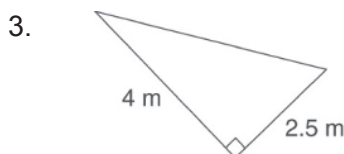
Area of Triangles

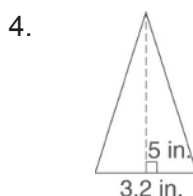
Practice and Problem Solving: C

Find the area of each triangle.









Solve.

5. If you wanted to find the total area of the triangles in Exercises 1, 2, and 4, what could you do?

6. A scale model of a street sign is in the shape of a triangle. The base is 4.25 centimeters and the height is 8.8 centimeters. What is the area of the street sign?

7. Rachel's earrings are in the shape of a triangle. The height of the earrings is $1\frac{1}{2}$ inches and the base is $\frac{3}{5}$ inch. What is the area of both of Rachel's earrings?

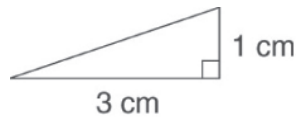
8. The face of a watch is in the shape of a triangle with a base of 6 centimeters and a height of 7.75 centimeters. What is the area of the face of the watch?

9. A flag in the shape of a triangle has an area of 25.2 square inches. The base of the flag is 6 inches. What is the height of the flag?

LESSON
13-2**Area of Triangles****Practice and Problem Solving: D**

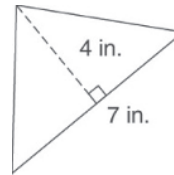
Find the area of each triangle. The first one is done for you.

1.

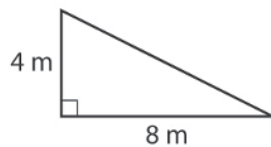


$$A = \frac{1}{2} \cdot 3 \cdot 1 = 1.5 \text{ cm}^2$$

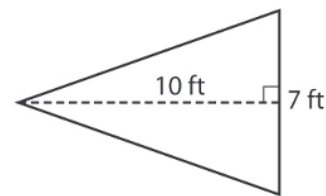
2.



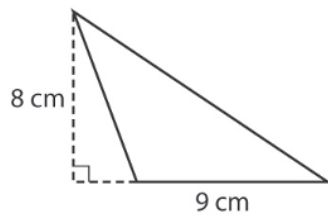
3.



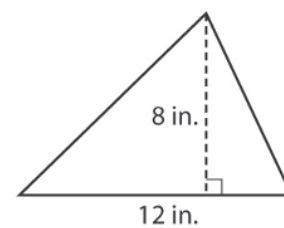
4.



5.



6.



Solve each problem. The first one is done for you.

7. A triangular-shaped rug has a base of 8 feet and a height of 7 feet.

What is the area of the rug?

$$A = \frac{1}{2} \cdot 8 \cdot 7 = 28 \text{ ft}^2$$

8. The sail on a sailboat is in the shape of a triangle that has a base of 12 feet and a height of 14 feet. What is the area of the sail?
-

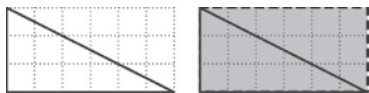
9. The front view of a square pyramid is in the shape of a triangle that has a base of 30 yards and a height of 40 yards. What is the area of the front view of the square pyramid?
-

LESSON
13-2

Area of Triangles

Reteach

To find the area of a triangle, first turn your triangle into a rectangle.



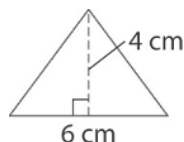
Next, find the area of the rectangle. $6 \cdot 3 = 18$ square units

The triangle is half the area of the formed rectangle or $A = \frac{1}{2}bh$, so divide the product by 2.

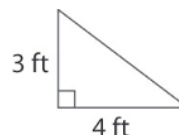
$18 \div 2 = 9$ So, the area of the triangle is 9 square units.

Find the area of each triangle.

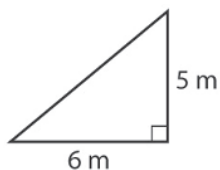
1.



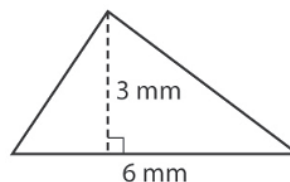
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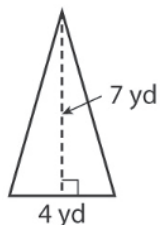
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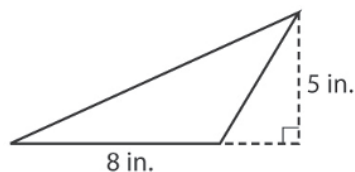
4.



5.

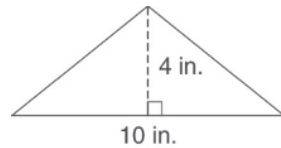


6.



LESSON
13-2**Area of Triangles****Reading Strategies: Follow a Procedure**

You can follow a procedure to help you find the area of a triangle.



Step 1: Use the formula $A = \frac{1}{2}bh$.



Step 2: Substitute the length of the base for b .



Step 3: Substitute the length of the height for h .



Step 4: Multiply.

Answer each question.

1. What is the first step in finding the area of the triangle?

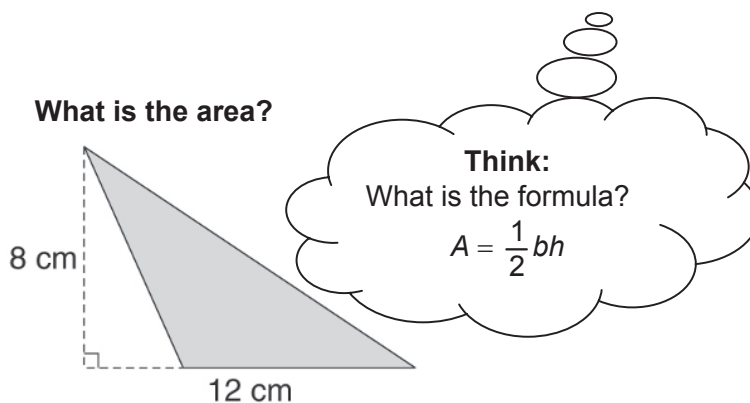
2. What are the second and third steps in finding the area of the triangle?

3. What is the area of the triangle?

4. What is the area of a triangle with base 18 meters and height 6 meters?

5. What is the area of a triangle with base 3.6 feet and height 2.5 feet?

6. How would you vary this procedure if you were given the area and base of a triangle and asked to find its height?

LESSON
13-2**Area of Triangles****Success for English Learners****Problem 1****Remember:**

A = Area b = base h = height
Examples of square units: ft^2 , yd^2 , cm^2

The base is 12 cm.

The height is 8 cm.

$$A = \frac{1}{2}bh \quad \text{Use formula.}$$

$$A = \frac{1}{2}(12 \cdot 8) \quad \text{Use measurements.}$$

$$A = \frac{1}{2}(96) \quad \text{Multiply. } 12 \cdot 8 = 96$$

$$A = \frac{1}{2}(96) = 48 \quad \text{Divide. } 96 \div 2 = 48$$

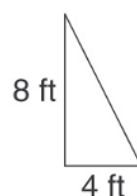
$$A = 48 \text{ cm}^2$$

So, the area is 48 cm^2 .

1. Does it matter which side of the triangle you use as the base and which you use as the height? Explain.

2. Why is the expression $\left(\frac{1}{2} \cdot 12\right) \cdot 8$ the same as $\frac{1}{2} \cdot (12 \cdot 8)$?

3. Find the area of the triangle shown.



LESSON
13-3**Solving Area Equations*****Practice and Problem Solving: A/B*****Solve.**

1. The front of an A-frame house is in the shape of a triangle. The height of the house is 20 feet. The area of the front of the A-frame is 600 square feet. Write and solve an equation to find the base of the A-frame house.
- _____

2. A countertop is in the shape of a trapezoid. The lengths of the bases are $70\frac{1}{2}$ and $65\frac{1}{2}$ inches long. The area of the countertop is 1,224 square inches. Write and solve an equation to find the height of the countertop.
- _____

3. The top of a coffee table is in the shape of a rectangle. The length of the top of the coffee table is 3.5 feet and the area is 10.5 square feet. What is the width of the top of the coffee table?
- _____

4. Jacob made a banner for a sporting event in the shape of a parallelogram. The area of the banner is $127\frac{1}{2}$ square centimeters. The height of the banner is $4\frac{1}{4}$ centimeters. What is the base of the banner?
- _____

5. McKenzie has enough paint to paint 108 square feet. She wants to paint her garage door, which has a height of 12 feet. The garage door is in the shape of a rectangle. If McKenzie has just enough paint to cover the garage door, what is the width of the door?
- _____

LESSON
13-3**Solving Area Equations*****Practice and Problem Solving: C*****Solve.**

1. The front of a podium is in the shape of a trapezoid with base lengths 4 and 8.5 feet. The height is 2 feet. A gallon of paint covers about 350 square feet. How many front frames of a podium can Lillian paint with 2 gallons of paint?

2. Kenneth's back yard is in the shape of a rectangle with a length of 12 yards and a width of 10 yards. A bag of grass seed costs \$25.99 and covers 400 square feet. How much will Kenneth spend on grass seed to cover his back yard?

3. The area of a triangular piece of stained glass is 50 square centimeters. If the height of the triangle is four times the base, how long are the height and base of the piece of stained glass?

4. A park is in the shape of a parallelogram. The park has an area of $776\frac{1}{4}$ square yards. The base of the park is $34\frac{1}{2}$ yards. Marta wants to jog 10 sprints. Each sprint is the same distance as the height of the park. How far will Marta sprint?

5. A quilt contains cuts of congruent right triangular pieces with a base of $8\frac{1}{2}$ centimeters and a height of $8\frac{1}{2}$ centimeters. How many triangular pieces are needed to make a rectangular quilt with an area of 4,335 square centimeters?

LESSON
13-3**Solving Area Equations*****Practice and Problem Solving: D***

Solve each problem. The first one is done for you.

1. Jennifer has a picture frame in the shape of a rectangle. The area of the picture frame is 35 square inches. The length of the picture frame is 7 inches. What is the width of the frame?

$$A = lw$$

$$35 = 7w$$

$$5 = w$$

5 in. _____

2. Christopher's back yard is in the shape of a trapezoid. The bases of his back yard are 30 and 40 feet long. The area of his back yard is 525 square feet. Write and solve an equation to find the height of Christopher's back yard.

3. Cindy made a triangular shaped sculpture with an area of 63 square inches. The height of the sculpture is 9 inches. What is the base length of the sculpture?

4. A floor mat is in the shape of a parallelogram. The mat has an area of 480 square inches. If the base of the mat is 24 inches, what is the height of the mat?

5. A trading token is in the shape of a trapezoid and has an area of 25 square centimeters. If the bases are 3 and 7 centimeters, what is the height of the token?

6. The back frame of a dog house is in the shape of a triangle with an area of 6 square feet. The height of the frame is 4 feet. What is the width of the frame?

LESSON
13-3**Solving Area Equations****Reteach**

You can use area formulas to find missing dimensions in figures.

The formula for area of a parallelogram is $A = bh$.

The formula for area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$.

The formula for area of a rhombus is $A = \frac{1}{2}d_1d_2$.

The formula for area of a triangle is $A = \frac{1}{2}bh$.

Suppose you know the area of a triangle is 28 square feet. You also know the length of the base of the triangle is 7 feet. What is the height of the triangle?

Use the formula for area of a triangle. $A = \frac{1}{2}bh$

Substitute known values. $28 = \frac{1}{2}(7)h$

Multiply both sides by 2. $56 = 7h$

Divide both sides by 7. $8 = h$

The height of the triangle is 8 feet.

Solve.

1. The area of a parallelogram is 150 square meters. The height of the parallelogram is 15 meters. What is the length of the parallelogram?

2. The length of one diagonal of a rhombus is 8 cm. The area of the rhombus is 72 square centimeters. What is the length of the other diagonal of the rhombus?

3. The area of a triangle is 32 square inches. The height of the triangle is 8 inches. What is the length of the base of the triangle?

4. The area of a rectangle is 34 square yards. The length of the rectangle is 17 yards. What is the width of the rectangle?

5. The area of a trapezoid is 39 square millimeters. The height of the trapezoid is 6 millimeters. One of the base lengths of the trapezoid is 5 millimeters. What is the length of the other base of the trapezoid?

LESSON
13-3**Solving Area Equations****Reading Strategies: Draw a Diagram**

You can find missing measurements of figures when you know the formula for the figure and when you are given other information about the figure.

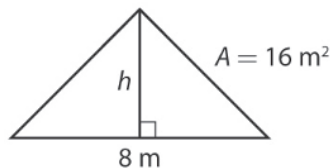
First, you need to know the different area formulas for common figures.

Figure	Area Formula
Parallelogram	$A = bh$
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$
Rhombus	$A = \frac{1}{2}d_1d_2$
Triangle	$A = \frac{1}{2}bh$

First, you should draw a diagram. Be sure to label the diagram with all the information you are given.

For example, a triangular-shaped poster has an area of 16 square meters and a base length of 8 meters. What is the height of the poster?

Draw a diagram. Label the diagram with the given information.



Now, use the given information with the formula for area of a triangle.

$$A = \frac{1}{2}bh \quad \text{Write the formula for area of a triangle.}$$

$$16 = \frac{1}{2}(8)(h) \quad \text{The area is 16 and the base is 8.}$$

$$16 = 4h \quad \text{Multiply } \frac{1}{2} \text{ and 8.}$$

$$4 = h \quad \text{Divide both sides by 4.}$$

The height of the poster is 4 meters.

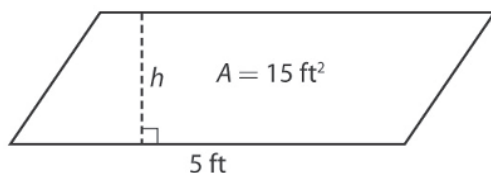
Solve.

1. A parallelogram has an area of 60 square inches. If the base of the parallelogram is 12 inches, what is the height of the parallelogram?

2. A trapezoid has an area of 45 square centimeters. If the bases are 10 and 5 centimeters, what is the height of the trapezoid?

LESSON
13-3**Solving Area Equations****Success for English Learners****Problem 1**

If you are given the length of the base and the area, you can find the height of a parallelogram.



$$A = bh$$

Write the formula for area.

$$15 = 5h$$

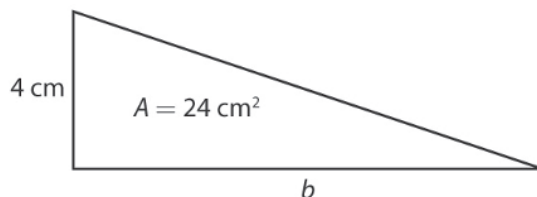
The area is 15 and the length of the base is 5.

$$3 = h$$

Divide both sides by 5.

Problem 2

If you are given the height and the area, you can find the length of the base of a triangle.



$$A = \frac{1}{2}bh$$

Write the formula for area.

$$24 = \frac{1}{2}(b)(4)$$

The area is 24 and the height is 4.

$$24 = 2b$$

Multiply $\frac{1}{2}$ and 4.

$$12 = b$$

Divide both sides by 2.

1. When given the area and known dimensions, what is the first step in finding an unknown measure in a figure?

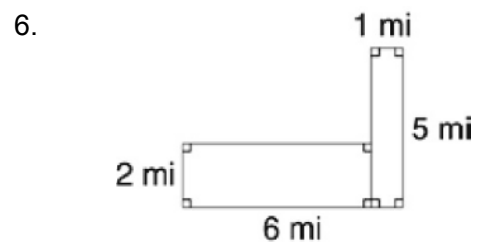
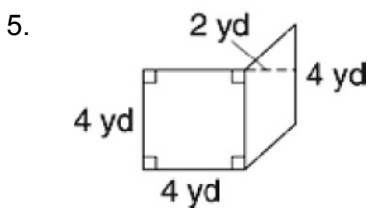
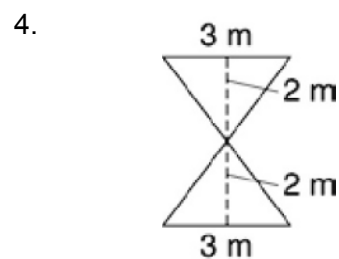
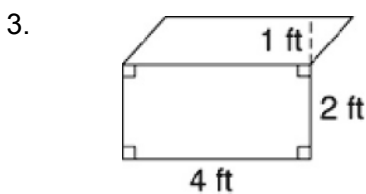
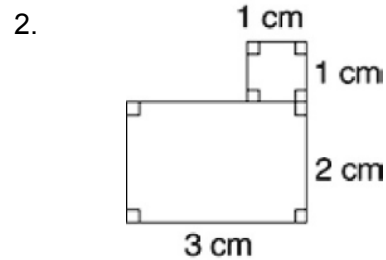
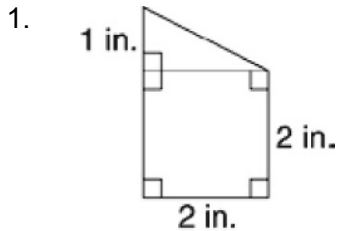
2. What is the next step?

LESSON
13-4

Area of Polygons

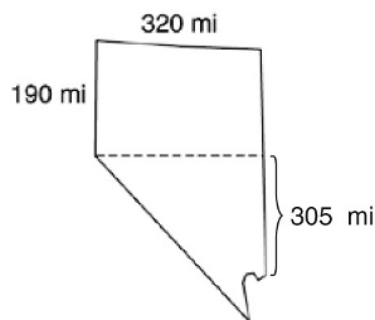
Practice and Problem Solving: A/B

Find the area of each polygon.

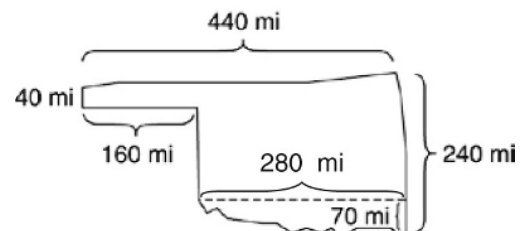


Solve.

7. The shape of Nevada can almost be divided into a perfect rectangle and a perfect triangle. About how many square miles does Nevada cover?



8. The shape of Oklahoma can almost be divided into 2 perfect rectangles and 1 triangle. About how many square miles does Oklahoma cover?



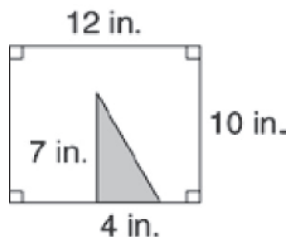
LESSON
13-4

Area of Polygons

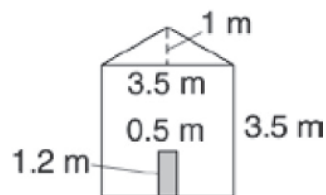
Practice and Problem Solving: C

Find the area of each figure. The shaded parts are cut out of the figures.

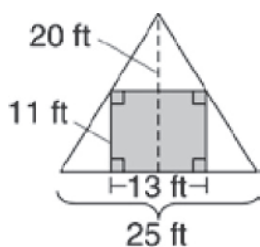
1.



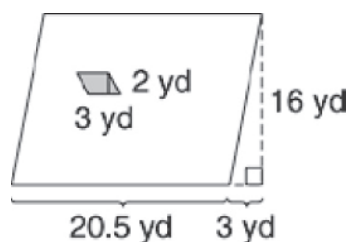
2.



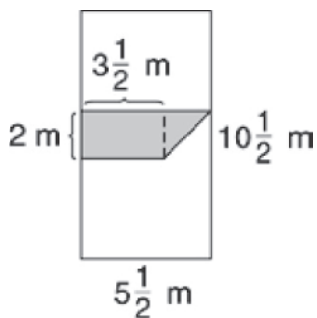
3.



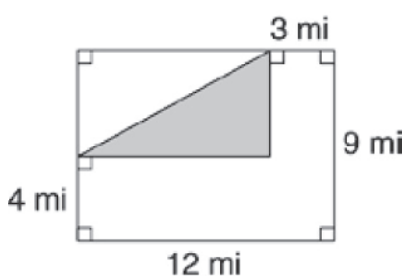
4.



5.



6.



Solve.

7. The front wall of the shed that Rita built is 9 feet tall and $5\frac{1}{4}$ feet wide.

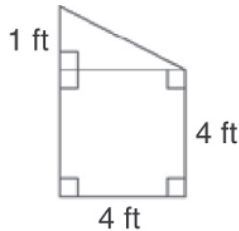
After she cut out a section for a door, the area of that wall was $26\frac{1}{4}$ square feet. The measurements of the doorway were whole numbers of feet. What size is the doorway?

8. The perimeter of a rectangular playground is 40 yards. It is $1\frac{1}{2}$ yards longer than it is wide. What is the area of the playground?

LESSON
13-4**Area of Polygons****Practice and Problem Solving: D**

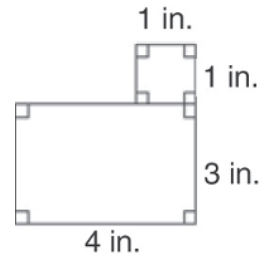
Find the area of each polygon. The first one is done for you.

1.

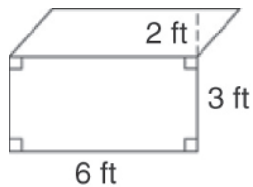


$$\frac{1}{2}(1 \times 4) + (4 \times 4) = 2 + 16 = 18 \text{ ft}^2$$

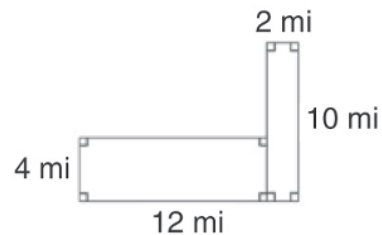
2.



3.



4.



Solve.

5. A rectangular painting is made up of two congruent squares with sides that are 2 feet long. What is the area of the entire painting?

6. A carpet is made up of two congruent triangles. The base of each triangle is 3 meters long, and the height is 6 meters. What is the area of the entire carpet?

LESSON
13-4

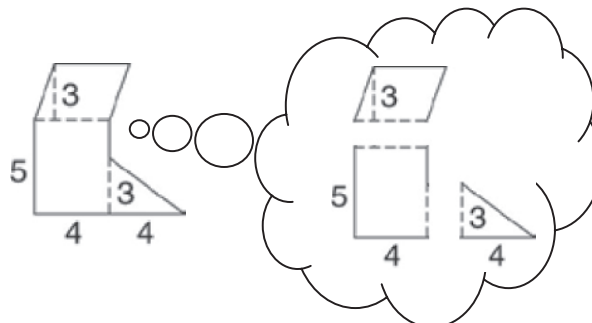
Area of Polygons

Reteach

Sometimes you can use area formulas you know to help you find the area of more complex figures.

You can break a polygon into shapes that you know. Then use those shapes to find the area.

The figure at right is made up of a triangle, a parallelogram, and a rectangle.



Triangle

$$\begin{aligned}
 A &= \frac{1}{2}bh \\
 &= \frac{1}{2}(3 \times 4) \\
 &= 6 \text{ square units}
 \end{aligned}$$

Parallelogram

$$\begin{aligned}
 A &= bh \\
 &= 3 \times 4 \\
 &= 12 \text{ square units}
 \end{aligned}$$

Rectangle

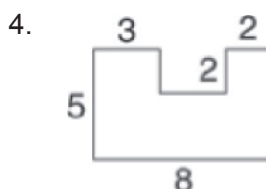
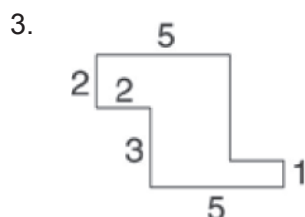
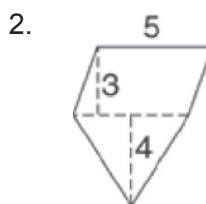
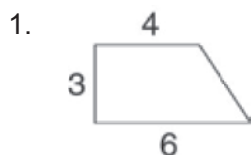
$$\begin{aligned}
 A &= lw \\
 &= 4 \times 5 \\
 &= 20 \text{ square units}
 \end{aligned}$$

Finally, find the sum of all three areas.

$$6 + 12 + 20 = 38$$

The area of the whole figure is 38 square units.

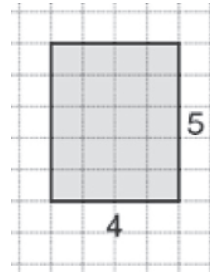
Find the area of each figure.



LESSON
13-4**Area of Polygons****Reading Strategies: Use a Flowchart**

To find the area of a rectangle, multiply its length by its width.

$$\begin{aligned} A &= l \cdot w \\ &= 4 \cdot 5 \\ &= 20 \text{ square units} \end{aligned}$$



If a polygon is made up of more than one rectangle, you can find its area by following the steps in the flowchart below.

Step 1: Separate the polygon into rectangles.



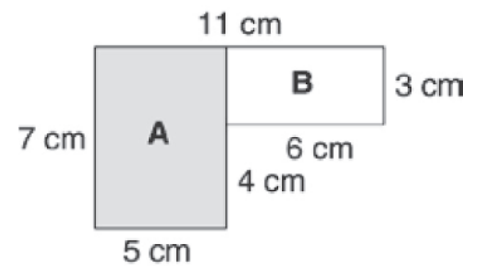
Step 2: Label the length and width of each rectangle.



Step 3: Find the area of each rectangle.



Step 4: Find the sum of all the areas.



Answer the questions. Refer to the six-sided polygon above.

1. What are the length and width of rectangle A? _____
2. What is the formula for the area of a rectangle? _____
3. What is the area of rectangle A? _____
4. What is the area of rectangle B? _____
5. What do you do with the areas of rectangles A and B to find the area of the polygon?

6. What is the area of the polygon? _____

LESSON
13-4

Area of Polygons

Success for English Learners

Problem 1

Find the area.

Separate the two rectangles.

1.8 cm 1.5 cm

2 cm 0.5 cm

$A_1 = 1.8 \cdot 1.5$
 $A_1 = 2.7$
 Area of the shape = $A_1 + A_2$
 $A = 2.7 + 1$
 $A = 3.7 \text{ cm}^2$

Add the area of each shape.

$A_2 = 2 \cdot 0.5$
 $A_2 = 1$

Remember to write the units.

Problem 2

Find the area.

$$A_1 = 8 \cdot 10 \quad A_2 = \frac{1}{2} \cdot 8 \cdot 3$$

$$A_1 = 80 \quad A_2 = 12$$

$$A (\text{shape}) = 80 + 12$$

$$A (\text{shape}) = 92 \text{ cm}^2$$

h = 13 cm

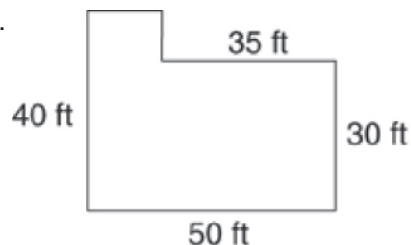
8 cm 8 cm 3 cm

10 cm

Separate the triangle from the rectangle.

Find the area.

1.

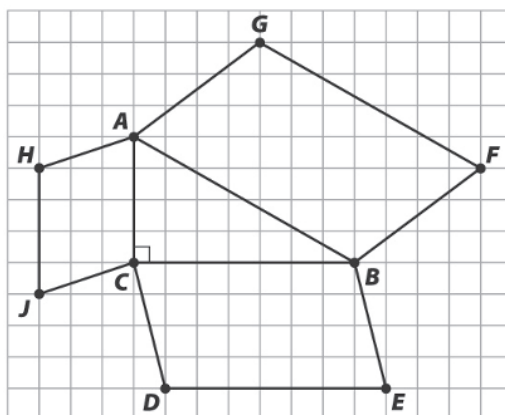


MODULE
13

Area and Polygons

Challenge

Answer the questions about the figure. Explain your answers and show your work.



1. What are the areas of parallelograms $ACJH$ and $BCDE$? (*Hint: Use the grid to find the areas.*)

2. What is the sum of the areas of parallelograms $ACJH$ and $BCDE$?

3. Draw auxiliary lines to form triangles inside or outside of parallelogram $ABFG$. How does the area of $ABFG$ compare to the sum of the areas of parallelograms $ACJH$ and $BCDE$?

4. Use the Pythagorean Theorem, $a^2 + b^2 = c^2$, to find the length of side AB of right triangle ABC .

5. How does the length of AB affect the area of $ABFG$ as it relates to the sum of the areas of parallelograms $ABFG$, $ACJH$, and $BCDE$?

LESSON
14-1

Distance in the Coordinate Plane

Practice and Problem Solving: A/B

Name the coordinates of each reflection.

1. Point *A* across the *x*-axis

New point: (_____, _____)

2. Point *B* across the *y*-axis

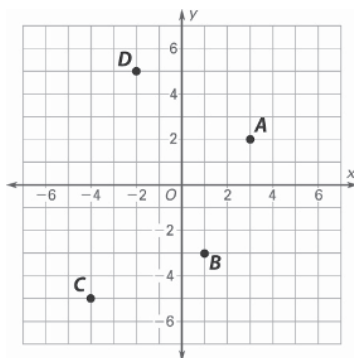
New point: (_____, _____)

3. Point *C* across the *x*-axis

New point: (_____, _____)

4. Point *D* across the *y*-axis

New point: (_____, _____)



Name the coordinates of each reflection of the given point.

5. $M(-2, -6)$

Across the *y*-axis: (_____, _____)

Across the *x*-axis: (_____, _____)

6. $N(4, 1)$

Across the *x*-axis: (_____, _____)

Across the *y*-axis: (_____, _____)

Find the distance between the points.

7. *A* and *B*: _____

8. *A* and *C*: _____

9. *B* and *D*: _____

10. *C* and *G*: _____

11. *D* and *F*: _____

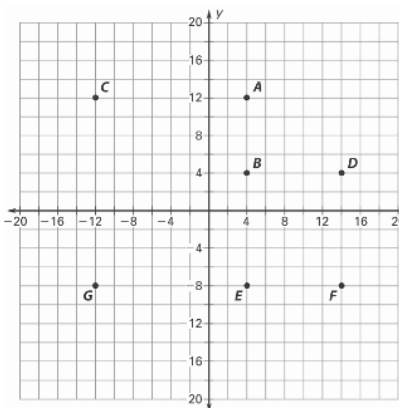
12. *E* and *F*: _____

13. *E* and *B*: _____

15. *E* and *G*: _____

14. *E* and *A*: _____

16. *F* and *G*: _____



Solve.

17. A taxi travels 25 kilometers east of an airport. Then, it travels from that point to a point that is 40 kilometers west of the airport. Finally, the taxi returns to the airport. How far did the taxi travel? Show your work.

LESSON
14-1**Distance in the Coordinate Plane*****Practice and Problem Solving: C***

Name the coordinates of the point that results from the transformations described.

1. Start at
- $(4, 7)$
- .

Reflect the point across the x -axis.Reflect again across the y -axis.

Coordinates: (_____, _____)

2. Start at
- $(-3, -5)$
- .

Reflect the point across the y -axis.Reflect again across the x -axis.

Coordinates: (_____, _____)

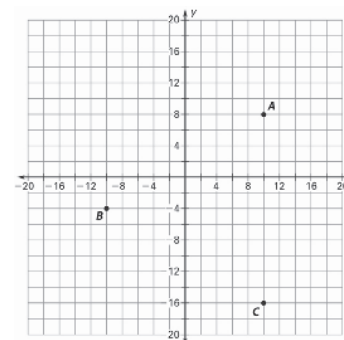
If a and b are whole numbers, what is the distance between the two points? Show your work.

- 3.
- $X(a, -b)$
- and
- $Y(a, 5b)$

- 4.
- $C(-3a, 3b)$
- and
- $D(-6a, 3b)$

Use the graph to answer the questions.

5. Explain why points
- A
- and
- B
- do
- not**
- result from reflections of the same point across the
- x
- axis and
- y
- axis, respectively.

_____

6. Show how point
- B
- can be reflected across the
- y
- axis so that it bisects the distance between points
- A
- and
- C
- .

7. What is the shortest distance from point
- B
- to the midpoint of the line between points
- A
- and
- C
- ?

8. The distance of point
- A
- to the
- x
- axis is given by the expression
- $4a$
- . The distance of the reflection of point
- A
- across the
- x
- axis to the
- y
- axis is given by the expression
- $3b$
- . What is the total distance from point
- A
- to its reflection across the
- x
- axis
- and*
- the
- y
- axis? Explain.

LESSON
14-1

Distance in the Coordinate Plane

Practice and Problem Solving: D

Use the coordinate plane for Exercises 1–5. The first one is done for you.

1. The x-coordinate of point X is 3.

The y-coordinate of point X is 3.

2. Reflect point X across the x-axis. What are the coordinates of the reflected point?

x-coordinate: _____

y-coordinate: _____

3. Reflect point X across the y-axis. What are the coordinates of the reflected point?

x-coordinate: _____ y-coordinate: _____

4. The x-coordinate of point Y is _____.

The y-coordinate of point Y is _____.

Reflect point Y across the x-axis. Give coordinates of the reflected point.

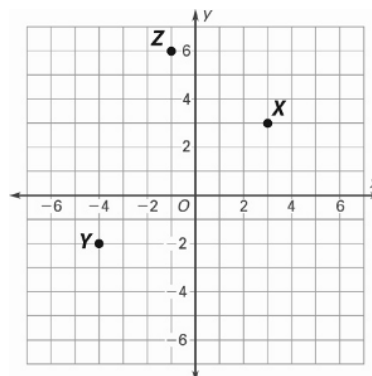
x: _____; y: _____

5. The x-coordinate of point Z is _____.

The y-coordinate of point Z is _____.

Reflect point Z across the x-axis. Give the coordinates of the reflected point.

x: _____; y: _____



Use the coordinate plane for Exercises 6–8. Find the distance between the points. The first one is done for you.

6. point A and point B

What is the x-coordinate of point A? -2

What is the x-coordinate of point B? 4

Subtract the x-coordinates. $|4 - (-2)| = 6$

The distance is 6 units.

7. point B and point C

The y-coordinate of point B is _____.

The y-coordinate of point C is _____.

Subtract the y-coordinates.

 - =

The distance is _____ units.

8. point C and point D

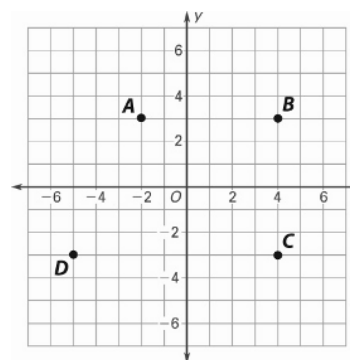
The x-coordinate of point C is _____.

The x-coordinate of point D is _____.

Subtract the x-coordinates.

 - =

The distance is _____ units.



LESSON
14-1

Distance in the Coordinate Plane

Reteach

Reflecting a Point

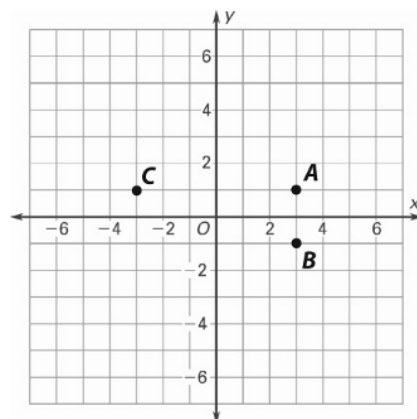
In this lesson, a point on a coordinate plane is reflected across the axes of the coordinate plane. The points B and C are reflections of point A across the x - and y -axes.

The coordinates of point A are $(3, 1)$.

Point B is the reflection of point A across the x -axis.

Point C is the reflection of point A across the y -axis.

The following rules can help you find the coordinates of a reflected point by looking at the signs of the coordinates.



Reflecting across the x -axis

“Reflect across x . \longrightarrow Change the y .”

In this example, point A 's x -coordinate, $+3$, stays the same when point A is reflected across the x -axis to become point B . Point A 's y -coordinate, $+1$, switches to -1 to become point B .

So, point B 's coordinates are $(3, -1)$.

Reflecting across the y -axis

“Reflect across y . \longrightarrow Change the x .”

In this example, point A 's y -coordinate, $+1$, stays the same when point A is reflected across the y -axis to become point C . Point A 's x -coordinate, $+3$, switches to -3 to become point C .

So, point C 's coordinates are $(-3, 1)$.

Name the coordinates of each point after it is reflected across the given axis.

1. $A(1, 3)$

x -axis

(____, ____)

2. $B(-4, 5)$

y -axis

(____, ____)

3. $C(6, -7)$

y -axis

(____, ____)

4. $D(-8, -9)$

x -axis

(____, ____)

Distance between Points

The distance between two points on a coordinate plane depends on whether their x - or y -coordinates are different. Look at the points on the grid above to solve the problems.

The distance between points A and B is the absolute value of the difference of the y -coordinates of the points.

The distance between points A and C is the absolute value of the difference of the x -coordinates of the points.

Find the distance between the two points.

5. points A and B

_____ units

6. points A and C

_____ units

LESSON
14-1

Distance in the Coordinate Plane

Reading Strategies: Use Graphic Aids

A coordinate plane can be used to illustrate a real-world problem. The example shows how the words in a problem can be translated into points and distances on the coordinate plane.

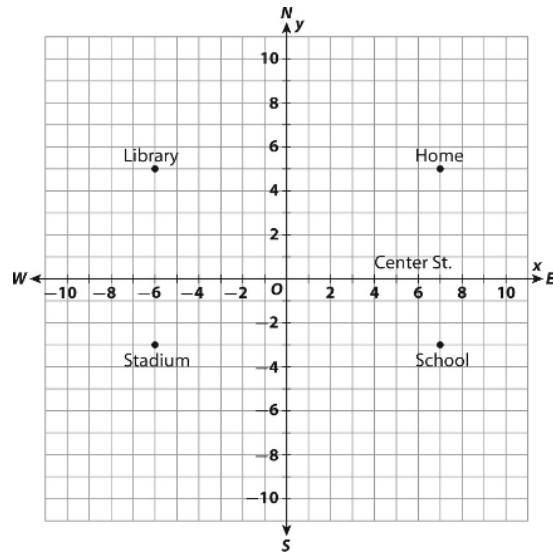
Example

The coordinate plane shows the location of a student's Home and School, and the town's Library and Stadium.

The x-axis is often used to represent east and west directions on city and road maps. The y-axis is often used to represent north and south directions.

Positive distances are often defined as being "east" or "north" of an agreed-upon "center," such as the origin on a coordinate plane.

Negative distances are defined as being "west" or "south" of the center.



Problem

How far north of School is Home?

Solution

"North" is the vertical distance between School and Home. School is 3 blocks south of Center Street; Home is 5 blocks north of Center Street.

To find the distance from Home to School count the blocks.

_____ blocks.

But the question asked was, "How far "north" of School is Home?" A more precise answer would be 8 blocks north.

Solve. Show your work.

- How far west of Home is the Library?

- How far south of the Library is the Stadium?

LESSON
14-1

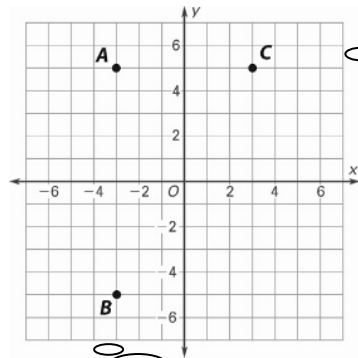
Distance in the Coordinate Plane

Success for English Learners

Problem 1

Which point is a reflection of point A across the y -axis?

Point B or Point C ?



What are the coordinates of point A ? **$(-3, 5)$**
 What are the coordinates of point C ? **$(3, 5)$**

What are the coordinates of Point B ?
 $(-3, -5)$

What changes when a point is reflected across the y -axis?

The x -coordinate.

What stays the same?

The y -coordinate.

Point C is the reflection of point A , across the y -axis because:

Point A 's x -coordinate changes to become point C 's x -coordinate: **-3 to $+3$.**

Point A 's y -coordinate stays the same to become point C 's y -coordinate: **$+5$.**

The answer is point C $(3, 5)$.

Problem 2

What is the distance between points A and C ?

Point A : **$(-3, 5)$**
 Point C : **$(3, 5)$**

Count the squares.
 Distance?

The distance from point A to point C is 6 units.

1. What kind of reflection of point A is represented by point B ?

2. What is the distance from point A to point B ?

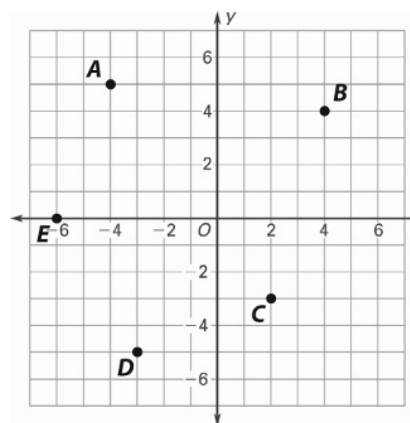
LESSON
14-2

Polygons in the Coordinate Plane

Practice and Problem Solving: A/B

List all of the polygons that can be formed by using some or all of the lettered vertices shown in the coordinate plane.

1. _____



Tell how many polygons can be formed by each set of points or set of points and a line.

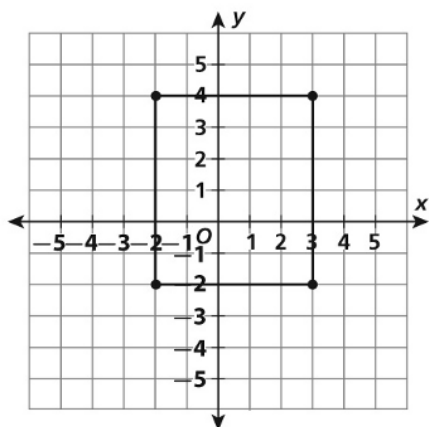
2. $(0, 1)$ and $(2, 3)$

3. $(4, 5)$, $(6, 7)$, and $(8, 9)$

4. $(3, 5)$ and the x-axis.

Find the perimeter and area of each polygon. Show your work.

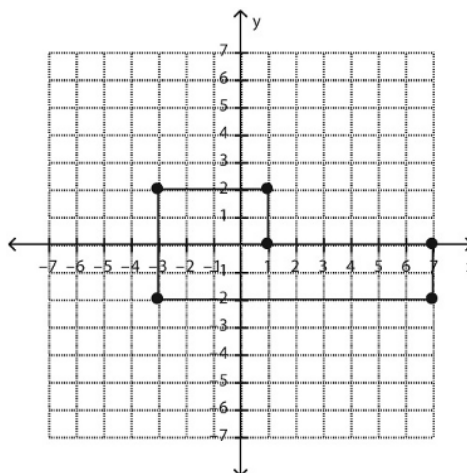
5.



Perimeter: _____

Area: _____

6.



Perimeter: _____

Area: _____

LESSON
14-2

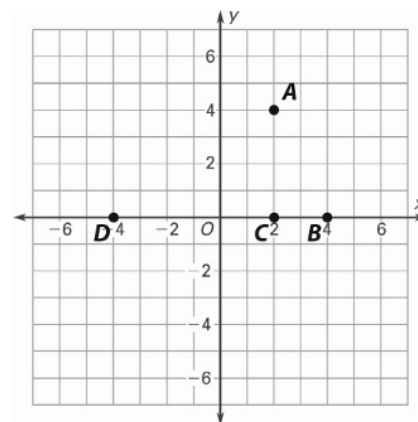
Polygons in the Coordinate Plane

Practice and Problem Solving: C

Use the coordinate plane for Exercises 1–11.

Write an expression for the perimeter of each triangle.
For the distance between points A and B , use x . For the distance between points A and D , use y .

1. triangle ABC _____
2. triangle ABD _____
3. triangle ACD _____



Answer each question.

4. What is an expression for the sum of the perimeters for triangles ABC and ACD ?

5. Compare your answer to Exercises 2 and 4. How does the perimeter of the largest triangle compare with the sum of the perimeters of the smaller triangles?

The formula for the area of a triangle is $A = \frac{1}{2}bh$, where b is the length of its base and h is the triangle's height. Any side of a triangle can be its base. The height is the vertical distance from the base to the vertex opposite the base. Find the areas of each triangle.

6. triangle ABC _____
7. triangle ACD _____
8. triangle ABD _____

Answer each question.

9. What is the sum of the areas of the triangles ABC and ACD ? _____
10. Compare your answers to Exercises 8 and 9. How does the area of the largest triangle compare with the sum of the areas of the smaller triangles?

11. Consider your answers to Exercises 5 and 10. How do the sums of the perimeters and the sum of the areas differ for these triangles?

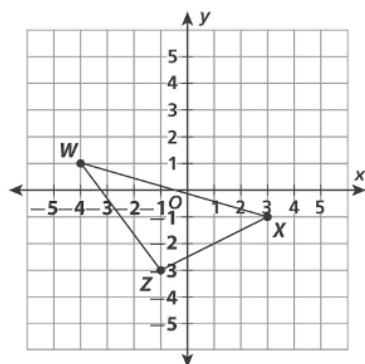
LESSON
14-2

Polygons in the Coordinate Plane

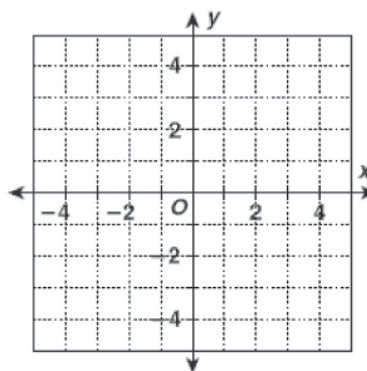
Practice and Problem Solving: D

Plot the points. Draw line segments to connect them to form a polygon. The first is done for you.

1. $W(-4, 1)$, $X(3, -1)$, $Z(-1, -3)$

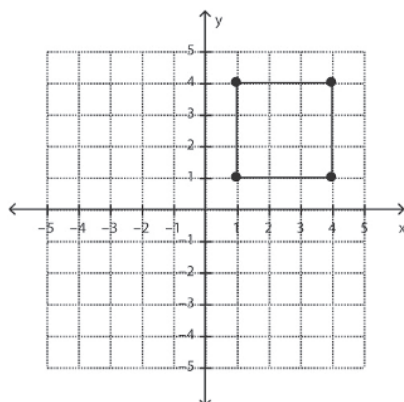


2. $A(-2, 3)$, $B(1, 3)$, $C(1, 0)$, $D(-2, 0)$



Find the area and perimeter of each figures. The first one is done for you.

3.



Name the coordinates of the vertices.

(1, 4), (4, 4), (4, 1), and (1, 1)

Find the lengths of the sides.

3, 3, 3, and 3

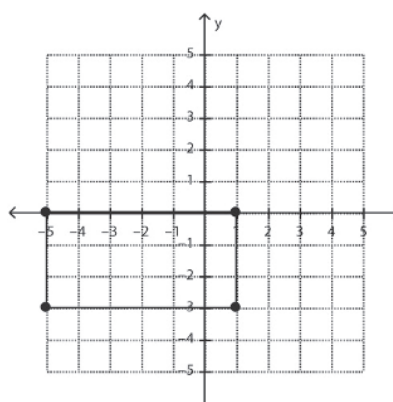
Find the perimeter.

$3 + 3 + 3 + 3 = 12$; 12 units

Find the area.

$3 \times 3 = 9$; 9 square units

4.



Name the coordinates of the vertices.

Find the lengths of the sides.

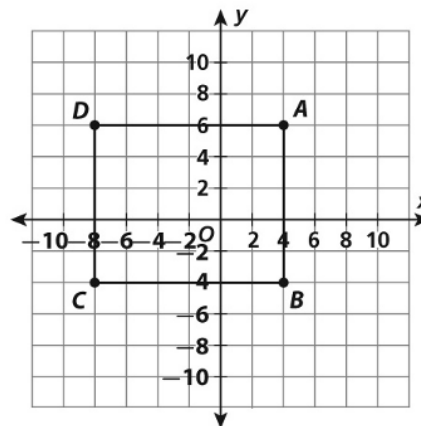
Find the perimeter.

Find the area.

LESSON
14-2**Polygons in the Coordinate Plane****Reteach**

Polygons are formed from three or more points, called *vertices*, that are connected by line segments and that enclose an area.

If the lengths of the sides are known, the area and perimeter of a polygon can be found. They can also be found if the coordinates of the vertices are known.

**Find the Perimeter**

First, identify the coordinates of the points that form the vertices of the polygon.

$A: (4, 6); B: (4, -4); C: (-8, -4); D: (-8, 6)$

Next, find the lengths of the sides.

$$AB = 10 \text{ units}$$

$$BC = 12 \text{ units}$$

$$CD = 10 \text{ units}$$

$$DA = 12 \text{ units}$$

Finally, add the lengths of the sides.

$$10 + 12 + 10 + 12 = 44$$

The perimeter of the polygon is 44 units.

Find the Area

First, identify the polygon. The figure is a rectangle, so its area is the product of its length and width.

Next, use the coordinates of the points to find the length and width.

$$AB = 10 \text{ units}$$

$$BC = 12 \text{ units}$$

Finally, multiply the length and width.

$$10 \times 12 = 120$$

The area of the polygon is 120 square units.

In this case, the area can also be found by counting the squares enclosed by the polygon. There are 30 squares.

How much area is represented by each square? 2×2 , or 4 square units.

The area is 30 cubes \times 4, or 120 square units.

Find the perimeter and area of the polygon enclosed by the points.

1. $(8, 6), (2, 6), (8, -5), \text{ and } (2, -5)$

Side lengths: _____

Perimeter: _____

Area: _____

2. $(0, 0), (0, 7), (7, 7), \text{ and } (7, 0)$

Side lengths: _____

Perimeter: _____

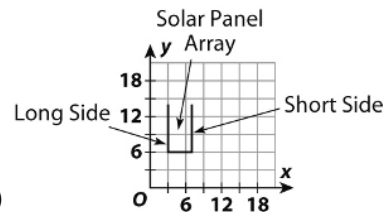
Area: _____

LESSON
14-2**Polygons in the Coordinate Plane****Reading Strategies: Analyze Information**

Real-world problems involving polygons can sometimes be solved more easily by sketching the polygon on a coordinate plane.

Example

There is a rectangular solar panel on the rectangular roof of an office building. The edges of the solar panel align with the edges of the roof. The short side of the solar panel is located 3 meters from one edge of the roof. The long side of the solar panel is located 6 meters from another edge of the roof. The diagram below shows one way of drawing the corner of the solar panel that is nearest to one corner of the roof. Let the vertex of the corner of the roof be the origin, $(0, 0)$, on a coordinate plane.

**Problem**

What are one possible set of coordinates for this corner of the solar panel? (*Hint: Remember, the corner of the building is $(0, 0)$.*)

Solution

The word *corner* is being used in place of the math term, *vertex*. The long side of the solar panel is 3 meters from one edge of the building. If we let the y -axis represent that side of the building, the x -coordinate of this corner of the solar panel is 3. The short side of the solar panel is 6 meters from one edge of the building. If we let the x -axis represent that side of the building, the y -coordinate of this corner of the solar panel is 6. So, the coordinates of this corner of the solar panel would be $(3, 6)$.

Note: Alternately, we could have drawn the diagram so that the short side of the solar panel is parallel instead to the x -axis. In that case, the coordinates of this corner of the solar panel would be reversed: $(6, 3)$.

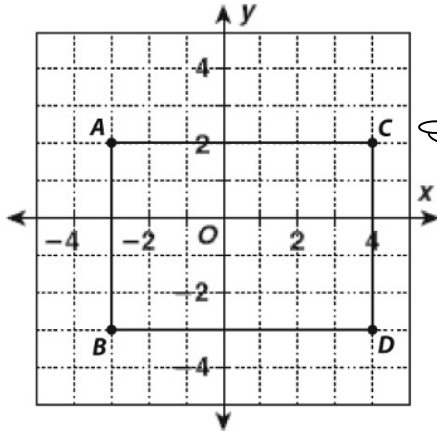
The long side of the solar panel measures 8 meters, and the short side measures 4 meters. Answer the questions.

1. As shown in the example, one corner of the solar panel has coordinates $(3, 6)$. What are the coordinates of the other corner of this side that is 3 meters from the edge? Show your work.

2. What are the coordinates of the other two corners of the solar panel? (*Hint: Find the length of the side of the array that is 6 meters from the edge of the roof as shown in the diagram.*) Show your work.

LESSON
14-2**Polygons in the Coordinate Plane****Success for English Learners****Problem 1**

What is the perimeter of rectangle $ABDC$?



How do you find perimeter?
Add the side lengths.

$$\begin{aligned}\text{Perimeter of rectangle } ABDC &= 5 + 7 + 5 + 7 \\ &= 24 \text{ units}\end{aligned}$$

Problem 2

Find the area of a rectangle using the sides.

What is the length?

The length is AC or BD . Why?

They are the same: **7 units**.

What is the width?

The width is AB or CD . Why?

They are the same: **5 units**.

How do you find the area?
Multiply length times width.

$$\begin{aligned}\text{Area of rectangle } ABDC &= 5 \times 7 \\ &= 35 \text{ square units}\end{aligned}$$

Answer the questions.

1. Why is the rectangle labeled $ABDC$, and not $ABCD$?

2. Find the area of $ABDC$ by counting the squares inside the polygon.

MODULE

14

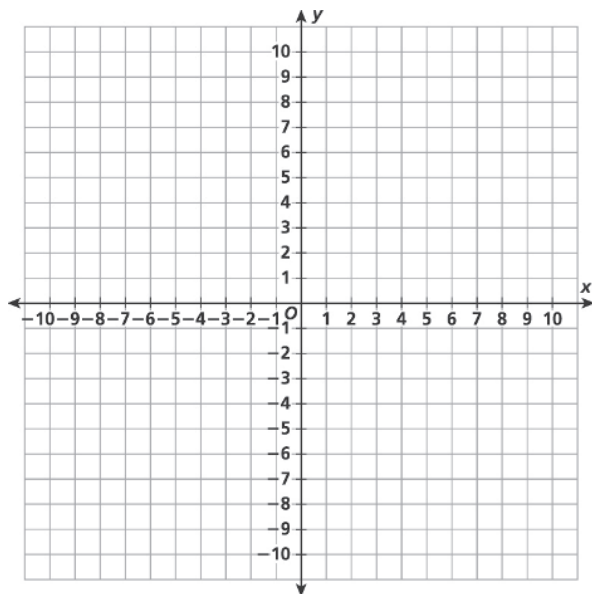
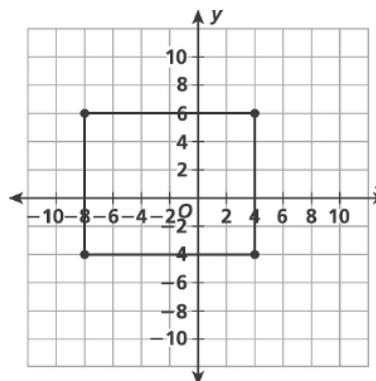
Distance and Area in the Coordinate Plane**Challenge**

Mrs. Chirag plans to have a new deck built on the back of her house. The coordinate plane shows plans for the deck in feet. The width of the deck along the x-axis has to be less than or equal to 20 feet.

The builder thinks there might be a better design for the deck with the same perimeter.

On the coordinate plane below, draw three different rectangular decks with the same perimeter as the original design.

*Note that the increments along the axes have been changed from those used on the original coordinate plane.



If the builder charges by the square foot, which design will cost the least to build? Explain your answer by answering the questions below.

1. Area of 1st design

2. Area of 2nd design

3. Area of 3rd design

4. Given the fact that the width has to be less than 20 feet, what are the dimensions of the deck that will result in the least area of all? Explain your answer.

5. In the real world, what dimensions of the deck with the given perimeter would be practical?

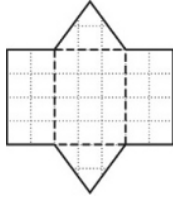
LESSON
15-1

Nets and Surface Area

Practice and Problem Solving: A/B

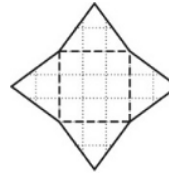
Find the surface area of each net.

1. Each square is one square meter.



_____ square meters

2. Each square is one square yard

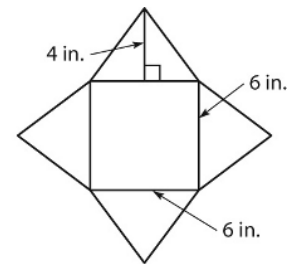
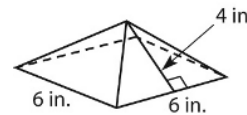


_____ square yards

3. A square pyramid has _____ square base
and _____ triangular faces.

Find its surface area.

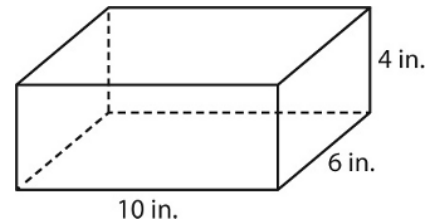
- The area of the base is _____ square centimeters.
- The area of the four faces is _____ square centimeters.
- The surface area is _____ square centimeters.



4. Josef makes wooden boxes for jewelry. He made 5 boxes like the one shown, and wants to cover all the outside faces with fabric.
- Find the surface area of one box.

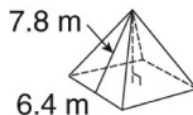
- Find the total surface area of 5 boxes.

- The fabric Josef is using comes in 100 square-inch pieces that cost \$6.25 each. What will his fabric cost?

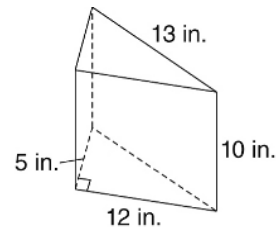


Calculate the surface area for each figure.

5. The base is a square.



- 6.



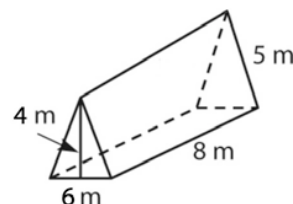
LESSON
15-1

Nets and Surface Area

Practice and Problem Solving: C

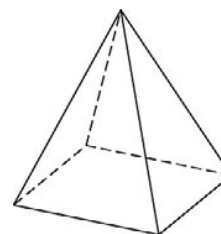
Solve.

1. What is the surface area of this isosceles triangular prism?
Show your work.



Surface area: _____

2. The base of the pyramid is square. What is its surface area if the pyramid's height is 4 feet and the area of the base is 36ft^2 ?
Show your work.



Surface area: _____

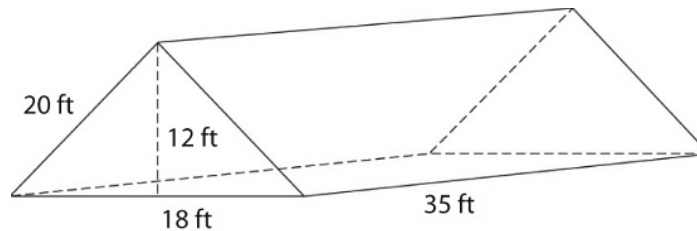
3. The top of the Washington Monument is a pyramid with a square base. Each triangular face is 58 feet tall and 34 feet wide and covered with white marble. About how many square feet of marble cover the faces of the pyramid?

4. A glass triangular prism for a telescope is 5.5 inches long. Each side of the prism's triangular bases is 4 inches long and about 3.46 inches high. About how much glass covers the surface of the prism?

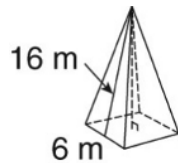
LESSON
15-1**Nets and Surface Area****Practice and Problem Solving: D**

Find the total surface area of the figures by completing the steps.
The first one is done for you.

1.

Height of the base: 12 feetLength of the base: 18 feetArea of the base: $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times \underline{18} \times \underline{12} = \underline{108}$ square feetNumber of bases \times area of a base = 2 \times 108 = 216 square feetArea of first face: 20 \times 35 = 700 square feetArea of second face: 20 \times 35 = 700 square feetArea of third face: 18 \times 35 = 630 square feetSurface area: 700 + 700 + 630 + 216 = 2246 square feet

2. The base is a square.



Height of a face: _____ meters

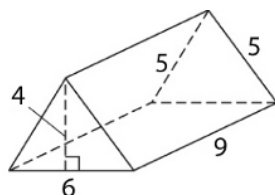
Base of a face: _____ meters

Area of a face: $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$ square metersNumber of faces \times area of a face = _____ \times _____ = _____ square metersArea of the base: _____ \times _____ = _____ square meters

Surface area: _____ + _____ = _____ square meters

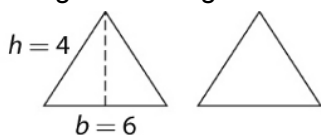
Complete.

3. The total surface area is the sum of the areas of the _____ and the
areas of the _____.

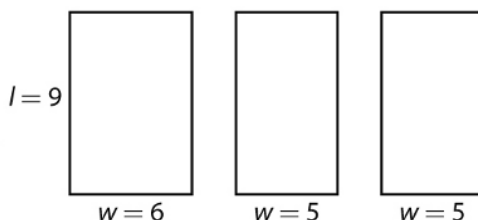
LESSON
15-1**Nets and Surface Area****Reteach**

To find the surface area of the regular triangular prism above, first find the area of each face or base.

2 congruent triangular bases



3 rectangular faces



$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(6 \cdot 4)$$

$$= 12 \text{ square units}$$

$$A = lw$$

$$= (9 \cdot 6)$$

$$= 54 \text{ square units}$$

$$A = lw$$

$$= (9 \cdot 5)$$

$$= 45 \text{ square units}$$

Then, find the sum of all of the faces of the prism.

$$SA = 12 + 12 + 54 + 45 + 45$$

$$= 168 \text{ square units}$$

The same procedure can be used to find the surface area of a **pyramid**.

The areas of the faces are added to the area of the base to give the total surface area.

Solve each problem.

1. A prism has isosceles triangle bases with leg lengths of 5 inches, 5 inches, and 8 inches, and a height of 3 inches. The distance between the bases is 12 inches. Find the surface area. Show your work.

2. A square pyramid has a base edge of 1 meter. The height of each triangular face is 1 meter. What is the pyramid's surface area? Show your work.

LESSON
15-1**Nets and Surface Area****Reading Strategies: Follow a Procedure**

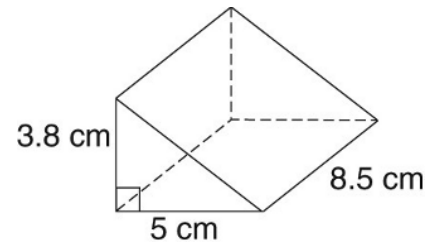
To find the surface area of a solid figure, follow a procedure. The procedures for a triangular prism and a triangular pyramid are different, but they share many of the same steps.

Step 1 The bases of the figure are right triangles, so one leg of the triangle is the base height, h , and the other leg is the base length, b :

$h = 3.8$ centimeters and $b = 5$ centimeters

Example

Find the lateral and total surface area.



Step 2 The area of the triangular base can be computed with the formula

for the area, A , of a triangle, which is $A = \frac{1}{2}bh$. Substitute the

values for b and h : $A = \frac{1}{2}(5)(3.8) = 9.5$ square centimeters

Step 3 Next, find the areas of the other faces. The first two are easy:

- For the first face, multiply 5 times 8.5, the length of the prism:
 $5 \times 8.5 = 42.5$ square centimeters
- For the second face, multiply 3.8 times 8.5:
 $3.8 \times 8.5 = 32.3$ square centimeters

Step 4 The third face requires that the Pythagorean Theorem be used to find the third side of the triangular base: $(3.8)^2 + 5^2 = x^2$

This gives $x^2 = 39.44$, or x is about 6.3 centimeters.

Multiply 6.3 by 8.5 to get the area of the third face: $6.3 \times 8.5 = 53.55$.

Step 5 Find the sum of the areas in Steps 3 and 4:

$42.5 + 32.3 + 53.55 = 128.35$ square centimeters.

Step 6 To find the total area, add the areas of the bases to Step 5:

$9.5 + 9.5 + 128.35 = 147.35$, or about 147 square centimeters.

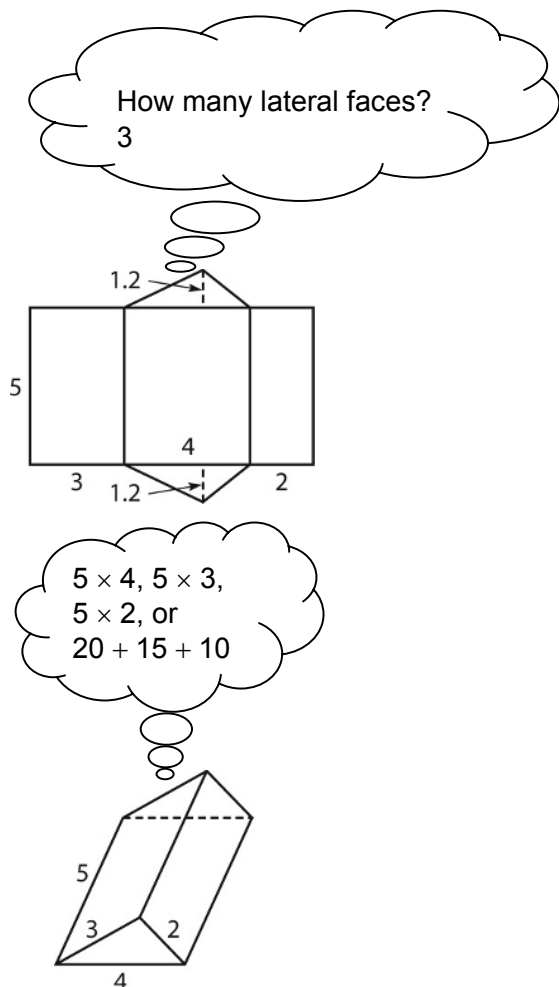
1. Modify the procedure above to find the surface area of a pyramid with a square base of area 16 square inches and a triangular face height of 5 inches. Show your work.

LESSON
15-1

Nets and Surface Area

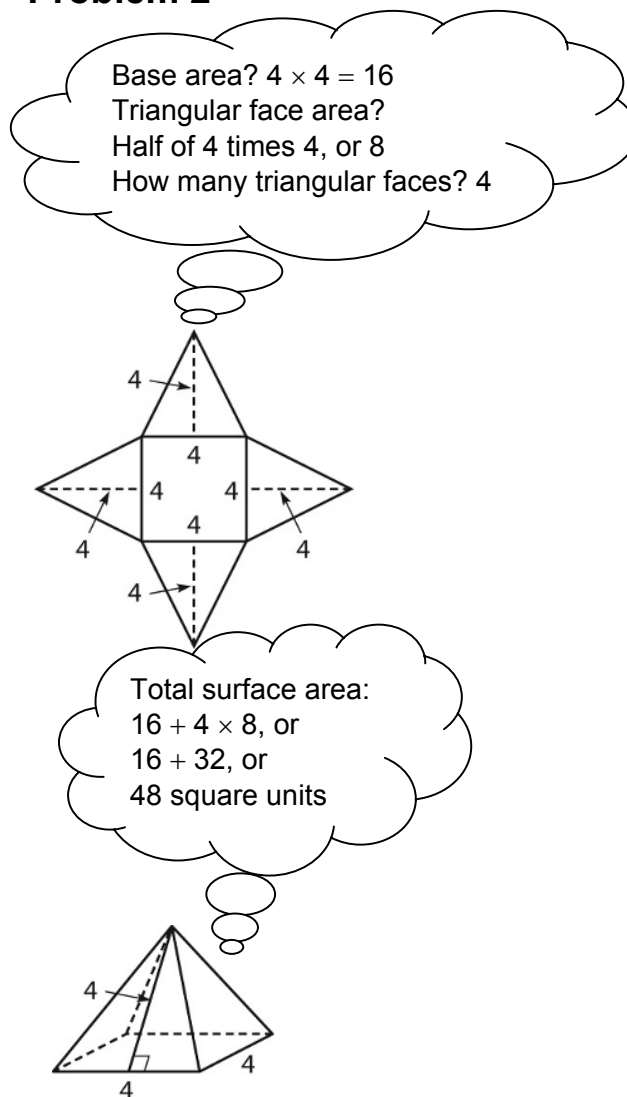
Success for English Learners

Problem 1



Lateral area = 45 square units

Problem 2



1. What measurements are needed to find the base area in Problem 1?

2. In Problem 1, find the base area. Show your work.

3. In Problem 2, how does the net help you find surface area?

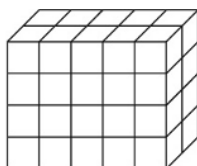
LESSON
15-2

Volume of Rectangular Prisms

Practice and Problem Solving: A/B

Use the formula for the volume of a rectangular solid to find the volume of each solid in cubic meters.

1.



5 cubes = 1 meter

Length:

_____ cubes = _____ meter

Width:

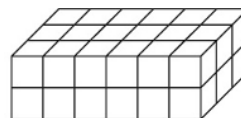
_____ cubes = _____ meter

Height:

_____ cubes = _____ meter

Volume: _____

2.



6 cubes = 1 meter

Length:

_____ cubes = _____ meter

Width:

_____ cubes = _____ meter

Height:

_____ cubes = _____ meter

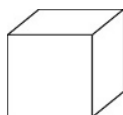
Volume: _____

Solve.

3. A student made a toy chest for his baby sister's square building blocks. Six layers of blocks can fit in the box, and each layer has 15 blocks. How many building blocks can the toy chest hold? Show your work.

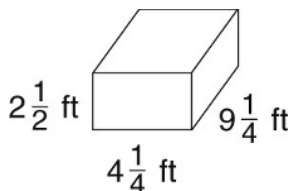
Find the volume of each figure. Show your work. Simplify your answers.

4.

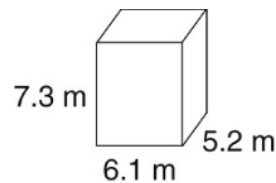


$s = 3$ in.

5.



6.



LESSON
15-2**Volume of Rectangular Prisms****Practice and Problem Solving: C**

In this lesson, there are several examples of rectangular solids that have heights, lengths, and widths that are less than one unit.

Example

The edges of a cube are each $\frac{1}{3}$ meter in length. What is its volume?

$$V = lwh$$

Because the figure is a cube, all three dimensions are the same.

So, $V = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$ cubic meter.

To find the dimensions of such prisms may require finding a cube root for a cube or dividing fractions for non-cubic rectangular prisms. Complete Table 1 for these solids with fractional dimensions.

Table 1: Rectangular Prism Sides and Volumes

Height	Length	Width	Volume
$\frac{1}{4}$ ft	$\frac{2}{3}$ ft	$\frac{3}{5}$ ft	1. _____ ft ³
0.1 m	2. _____ m	0.01 m	0.0005 m ³
$\frac{4}{3}$ in.	$\frac{3}{4}$ in.	1 in.	3. _____ in ³
1.1 km	4. _____ km	0.9 km	9.9 km ³

Use the data in Table 1 to complete Table 2 with $>$, $<$, or $=$.

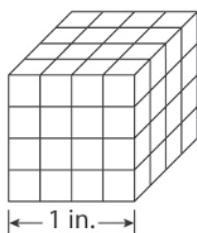
Table 2: Rectangular Prism Volumes

Height	Length	Width	Volume
$h < 1$	$l < 1$	$w < 1$	5. $V \bigcirc 1$
$h > 1$	$l > 1$	$w > 1$	6. $V \bigcirc 1$
h	$\frac{1}{h}$	$w = 1$	7. $V \bigcirc 1$

LESSON
15-2**Volume of Rectangular Prisms****Practice and Problem Solving: D**

Answer the questions to find the volume of each solid in cubic inches. The first one is started for you.

1.

How many cubes per inch? 4Height in cubes: 4Height in inches: 1

Width in cubes: _____

Width in inches: _____

Depth in cubes: _____

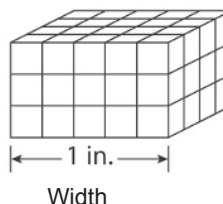
Depth in inches: _____

Volume = height \times width \times depth

Volume = _____ cubes

Volume = _____ cubic inches

2.



How many cubes per inch? _____

Height in cubes: _____

Height in inches: _____

Width in cubes: _____

Width in inches: _____

Depth in cubes: _____

Depth in inches: _____

Volume = height \times width \times depth

Volume = _____ cubes

Volume = _____ cubic inches

Solve. The first one is done for you.

3. The government plans to build a new dam shaped like a rectangular prism. The base is 1,224 feet long and 660 feet wide. The dam will be 726 feet high. Ignore the spaces within the dam that will be hollow to hold machinery. If the dam were made of solid concrete, how many cubic feet of concrete would be needed? Show your work.

$$V = l \times w \times h; V = 1,224 \times 660 \times 726 = 586,491,840 \text{ ft}^3 \text{ or about } 590,000,000 \text{ ft}^3$$

of concrete

4. The world's largest chocolate bar is a rectangular prism weighing more than a ton! The bar is 9 feet long, 4 feet tall, and 1 foot wide. How many cubic feet of chocolate does it contain? Show your work.

LESSON
15-2**Volume of Rectangular Prisms****Reteach**

The volume of a rectangular prism is found by multiplying its length, width, and height. In some cases, instead of the length and width, the area of one of the bases of the prism will be known.

Length, width, height, and volume

A rectangular prism has dimensions of 2.5 meters, 4.3 meters, and 5.1 meters. What is its volume to two significant figures?

Solution

$$V = l \times w \times h$$

$$V = 2.5 \times 4.3 \times 5.1$$

$$= 54.825$$

To two significant figures, the volume of the prism is 55 cubic meters.

Base area, height, and volume

A rectangular prism has a base area of $\frac{2}{3}$ of a square foot. Its height is $\frac{1}{2}$ foot.

What is its volume?

Solution

$$V = A_{\text{base}} \times h$$

$$V = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

The volume of the prism is $\frac{1}{3}$ cubic foot.

Find the volume of a rectangular prism with the given dimensions.

1. length: $\frac{2}{3}$ yd; width: $\frac{5}{6}$ yd; height: $\frac{4}{5}$ yd _____

2. base area: 12.5 m^2 ; height: 1.2 m _____

The density of a metal in a sample is the mass of the sample divided by the volume of the sample. The units are mass per unit volume.

Problem The mass of a sample of metal is 2,800 grams. The sample is in the shape of a rectangular prism that measures 5 centimeters by 7 centimeters by 8 centimeters. What is the volume of the sample?

$$V = 5 \times 7 \times 8$$

$$= 280 \text{ cm}^3$$

What is the density of the sample?

$$2,800 \div 280 = 10 \text{ g/cm}^3$$

3. A sample of metal has a mass of 3,600 grams. The sample is in the shape of a rectangular prism that has dimensions of 2 centimeters by 3 centimeters by 4 centimeters. What is the density of the sample?

LESSON
15-2**Volume of Rectangular Prisms****Reading Strategies: Identify Key Terms and Vocabulary**

To solve mathematical problems, it is important to identify and interpret terms and vocabulary.

Example

A cube has a volume of 27 cubic meters. A rectangular prism that is not a cube has a height that is twice the length of the edge of the cube. The volume of the rectangular prism is twice that of the cube. What other dimensions of the rectangular prism can be calculated?

Step 1 Start with what is given: A cube has a volume of 27 cubic meters. The key terms are “**cube**” and “**volume**.” Since the length, width, and height of a cube are all the same, the length of the edge of the cube can be calculated by taking the cube root of the volume. The cube root of 27 is 3.

Step 2 Move on to the second sentence of the problem: The rectangular prism has a height that is twice the length of the edge of the cube. The key terms here are “**height**” and “**twice**.” Since the cube has a side of 3 meters, the prism has a height of 2×3 , or 6.

Step 3 The third sentence has one key fact: The volume of the prism of “**twice**” the volume of the cube. Since the volume of the cube is 27 cubic meters, the volume of the prism is 2×27 , or 54 cubic meters.

Step 4 The last sentence contains the phrase “other **dimensions** of the prism.” To find those other dimensions requires the volume formula for the prism, $V = l \times w \times h$. Substitute the numbers found in Steps 2 and 3 into this formula.

$$54 = l \times w \times 6$$

Divide both sides of the formula by 6.

$$9 = l \times w$$

Since no other information is given about the dimensions l and w , the conclusion is that the area of the base of the prism is 9 square meters.

Identify some of the key terms and vocabulary in each problem.

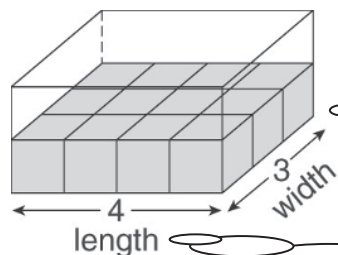
Then solve.

1. A cube-shaped container has twice the volume of a container shaped like a rectangular prism that has dimensions of 3 feet by 4 feet by 9 feet. What is the length of the cube’s edge?

2. The edge of a cube-shaped container is doubled. By how much is its volume increased?

LESSON
15-2**Volume of Rectangular Prisms****Success for English Learners****Problem 1**

The **volume** of a prism is the number of cubic units it holds.



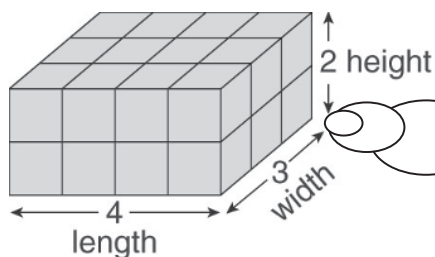
Look at the bottom layer of cubes in this rectangular prism.

How many cubes **long** is the bottom layer? **4**
How many cubes **wide** is the bottom layer? **3**

$$4 \times 3 = 12$$

So, there are **12 cubes** in the bottom layer of the prism.

Now, look at the top layer of cubes.



How many cubes **long** is the top layer? **4**
How many cubes **wide** is the top layer? **3**

$$4 \times 3 = 12$$

So, there are **12 cubes** in the top layer of the prism.

There are **12 cubes** in the bottom layer.

There are **12 cubes** in the top layer.

$$12 + 12 = 24$$

So, there are 24 cubes in the prism. The volume of the prism is 24 cubic units.

Solve.

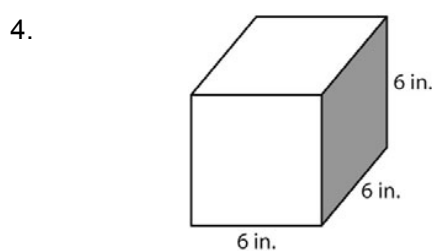
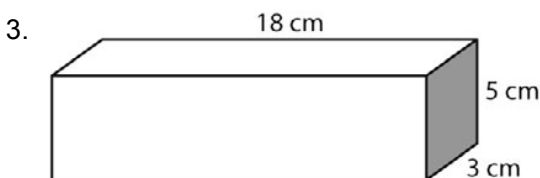
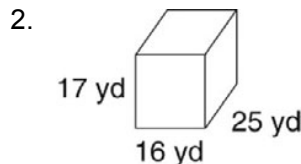
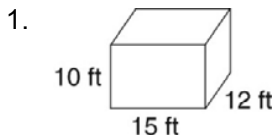
1. A rectangular prism is 5 cubes long, 4 cubes wide, and 3 cubes high.
What is the volume of this prism?

LESSON
15-3

Solving Volume Equations

Practice and Problem Solving: A/B

Find the volume of each figure.



Solve.

5. Fawn built a sandbox that is 6 feet long, 5 feet wide, and $\frac{1}{2}$ foot tall.
How many cubic feet of sand does she need to fill the box?
- _____

6. A pack of gum is in the shape of a rectangular prism with a length of 8 centimeters and width of 2 centimeters. The volume of the pack of gum is 48 cubic centimeters. What is the height of the pack of gum?
- _____

7. A block of cheese is in the shape of a rectangular prism with a width of 2.5 inches and a height of 5 inches. The volume of the block of cheese is 75 cubic inches. What is the length of the block of cheese?
- _____

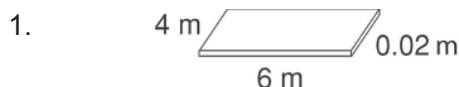
8. A tissue box is in the shape of a rectangular prism with a volume of 528 cubic inches. The length of the box of tissues is 12 inches and the height is $5\frac{1}{2}$ inches. What is the width of the box of tissues?
- _____

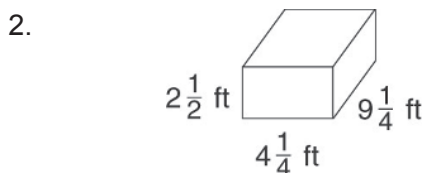
LESSON
15-3

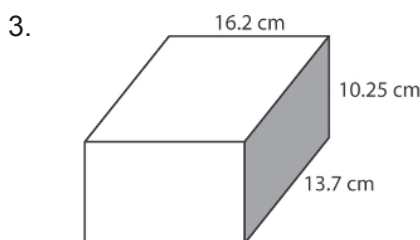
Solving Volume Equations

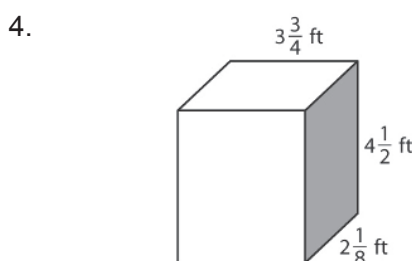
Practice and Problem Solving: C

Find the volume of each figure.









Solve.

5. If you changed the measures in Exercises 1 and 3 to fractions and mixed numbers, would you get the same volumes? _____

6. A rectangular prism's base is 8 feet long. It is $2\frac{1}{2}$ times taller than it is long and $\frac{1}{2}$ as wide as it is tall. What is the volume of that prism?

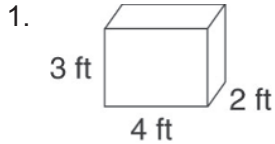
7. A box of mashed potato flakes has a volume of 220 in^3 . The box is 8 inches long and 11 inches tall. What is the width of the box of mashed potato flakes?

8. A building shaped like a rectangular prism is 42 yards long, 30 yards wide, and 120 yards tall. On average, it cost \$0.02 per cubic yard to provide heat and electricity for one month. What is the heat and electric bill for one month?

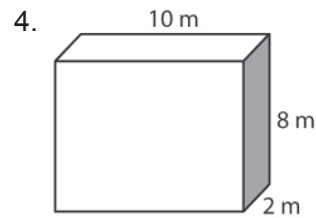
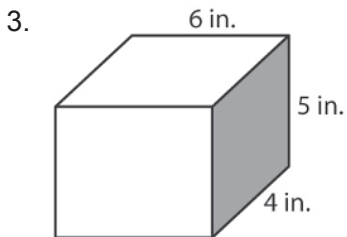
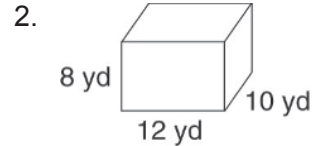
9. A 12 inches by 12 inches by 12 inches container of water was placed in the freezer. Water expands 4% when it is frozen. What is the volume of the container when it is frozen? Give your answer in cubic inches and cubic feet.

LESSON
15-3**Solving Volume Equations*****Practice and Problem Solving: D***

Find the volume of each figure. The first one is done for you.



$V = 3(4)(2) = 24$; 24 ft^3



Solve each problem. The first one is done for you.

5. Tim made a toy chest for his little sister's square building blocks. If 6 layers of blocks can fit in the box, and each layer has 15 blocks, how many building blocks can the toy chest hold in all?

$V = 15(6) = 90$; **90 blocks**

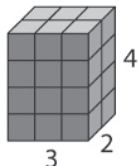
6. Kathy bought a jewelry box in the shape of a rectangular prism. The volume of the jewelry box is 192 cubic inches. The length and width of the jewelry box are 8 and 6 inches respectively. What is the height of the jewelry box?

7. A filing cabinet has a height of 4 feet and a length of 2 feet. The volume of the filing cabinet is 24 cubic feet. What is the width of the filing cabinet?

8. A box of business cards is in the shape of a rectangular prism. The volume of the box of cards is 360 cubic centimeters. The length of the box is 12 centimeters and the height of the box is 5 centimeters. What is the width of the box of business cards?

LESSON
15-3**Solving Volume Equations****Reteach**

Volume is the number of cubic units needed to fill a space. To find the volume of a rectangular prism, first find the area of the base.



length = 3 units

width = 2 units

$$A = lw = 3 \cdot 2 = 6 \text{ square units}$$

The area of the base tells you how many cubic units are in the first layer of the prism.

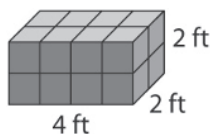
The height is 4, so multiply 6 by 4.

$$6 \cdot 4 = 24$$

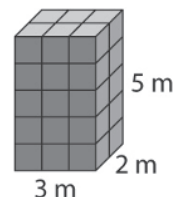
So, the volume of the rectangular prism is 24 cubic units.

Find each volume.

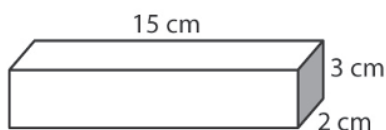
1.



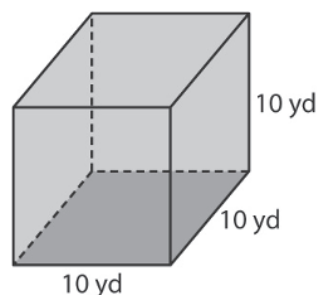
2.



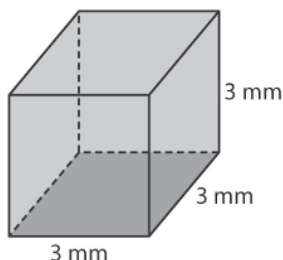
3.



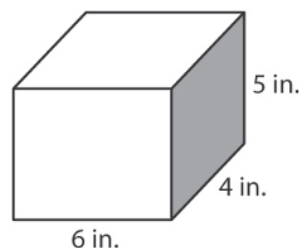
4.



5.



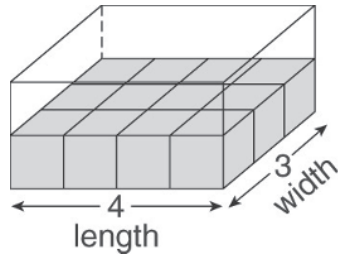
6.



LESSON
15-3**Solving Volume Equations****Reading Strategies: Analyze Information**

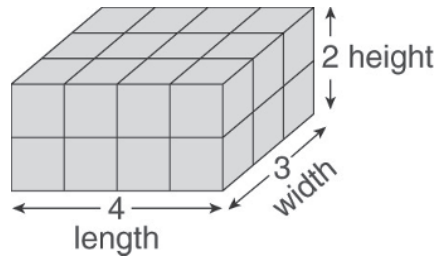
You can think of the **volume** of a prism as the number of unit cubes it contains.

Look at the first layer of cubic units in this rectangular prism.



1. How many cubes long is the first layer? _____
2. How many cubes wide is the first layer? _____
3. How many cubes are in the first layer? _____

Now look at the next layer of cubes.



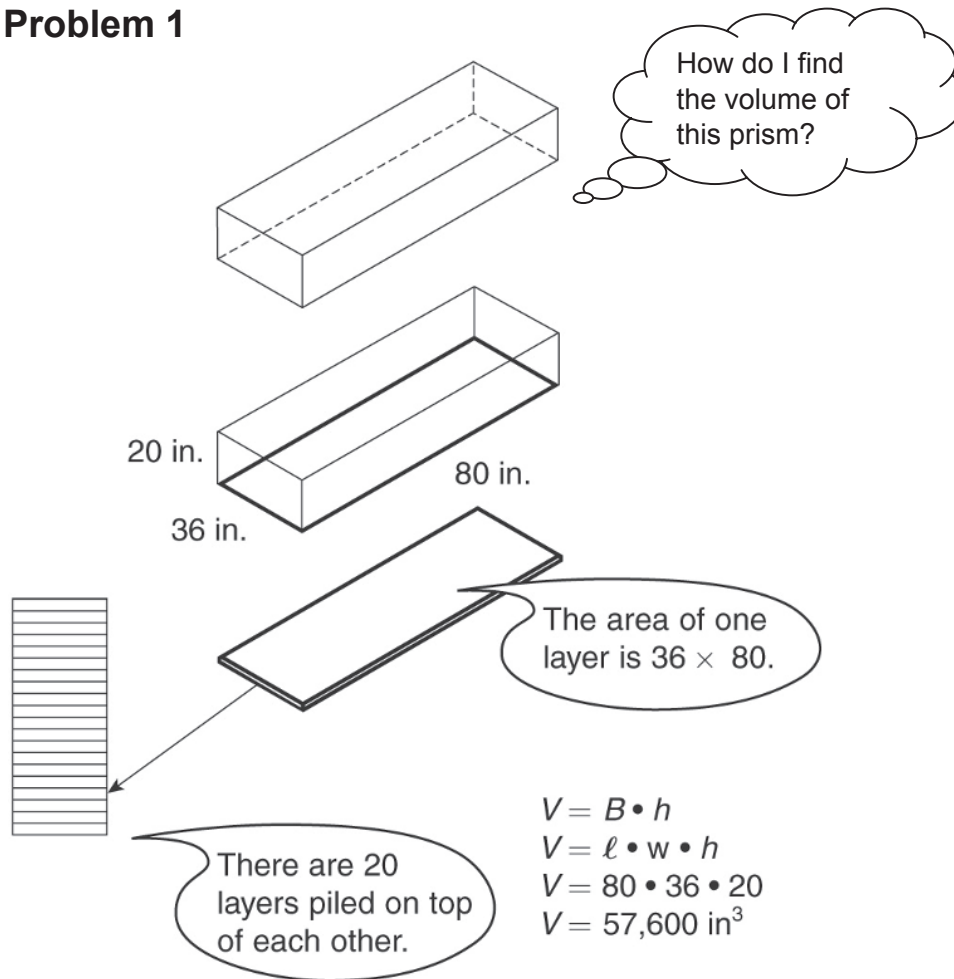
4. How many cubes long is the top layer? _____
5. How many cubes wide is the top layer? _____
6. How many cubes are in the top layer? _____
7. What is the total number of cubes in both layers? _____
8. What is the total volume of the rectangular prism? _____
9. Add another layer and find the volume for the new figure.
Show your work.

LESSON
15-3

Solving Volume Equations

Success for English Learners

Problem 1



1. Describe how to find the volume of a rectangular prism.

2. How are units for volume different from units for area?

MODULE
15**Surface Area and Volume of Solids****Challenge**

Which rectangular prism has the greatest surface area? Which rectangular prism has the greatest volume? The answers to both questions depend on the dimensions of the prisms you are comparing.

The surface area of a rectangular prism can be found by using the formula below, where h is a prism's height, l is its length, and w is its width.

$$S.A. = 2hl + 2lw + 2wh$$

The volume of a rectangular prism can be found by using the formula $V = l \times w \times h$.

Suppose the sum of the height, length, and width of a rectangular prism is 30 meters. The table shows three possible sets of length, width, and height whose sum is 30 meters. Complete the table. Then answer the questions that follow.

Height	Length	Width	S.A.	Volume
10 m	10 m	10 m	1. _____	2. _____
15 m	5 m	3. _____	4. _____	5. _____
20 m	6. _____	5 m	7. _____	8. _____

- Based on the data in the table, what conclusion can you draw about the shape of a rectangular prism that will yield the greatest volume?

- How does the surface area of the figure with the greatest volume compare to the surface areas of the other shapes?

- Describe the shape of the rectangular-prism boxes that are commonly used to package dry cereal and dry detergent.

- Are these boxes designed to hold the maximum amount of product? If not, why do you think the packaging has the shape it does?

