LESSON 7-1 Equations with the Variable on Both Sides

Practice and Problem Solving: A/B

Use algebra tiles to model and solve each equation.

1. \( x + 3 = -x - 5 \)
2. \( 1 - 2x = -x - 3 \)
3. \( x - 2 = -3x + 2 \)

Fill in the boxes to solve each equation.

4. \( 4a - 3 = 2a + 7 \)
   \[ \begin{array}{c}
   -2a \\
   +3 \\
   \end{array} \]
   \[ \begin{array}{c}
   2a - 3 = 7 \\
   +3 \\
   \end{array} \]
   \[ \begin{array}{c}
   2a = \ ] \\
   \end{array} \]
   \[ \begin{array}{c}
   a = \ ] \\
   \end{array} \]

5. \( 7x - 1 = 2x + 5 \)
   \[ \begin{array}{c}
   -2x \\
   +1 \\
   \end{array} \]
   \[ \begin{array}{c}
   5x = 6 \\
   \end{array} \]
   \[ \begin{array}{c}
   x = \ ] \\
   \end{array} \]

6. \( -3r + 9 = -4r + 5 \)
   \[ \begin{array}{c}
   +4r \\
   -9 \\
   \end{array} \]
   \[ \begin{array}{c}
   r + 9 = 5 \\
   -9 \\
   \end{array} \]
   \[ \begin{array}{c}
   r = \ ] \\
   \end{array} \]

Solve.

7. \( 3y + 1 = 4y - 6 \)
8. \( 2 + 6x = 1 - x \)
9. \( 5y + 4 = 4y + 5 \)

Write an equation to represent each relationship. Then solve the equation.

10. Ten less than 3 times a number is the same as the number plus 4.

11. Six times a number plus 4 is the same as the number minus 11.

12. Fifteen more than twice the hours Carla worked last week is the same as three times the hours she worked this week decreased by 15. She worked the same number of hours each week. How many hours did she work each week?
Solve.

1. \(-v + 5 + 4v = 1 + 5v + 3\)  
2. \(15 - x = 2(x + 3)\)  
3. \(5(r - 1) = 2(r - 4) - 6\)

4. \(6m - 11 = 2 + 9m - 1\)  
5. \(4(3x - 1) = 3 + 8x - 11\)  
6. \(-2(t + 2) + 5t = 6t + 11\)

Write an equation to represent each relationship. Then solve.

7. Twelve decreased by twice a number is the same as 8 times the sum of the number plus 4. What is the number?

8. Three added to 8 times a number is the same as 3 times the value of 2 times the number minus 1. What is the number?

9. Company A offers a starting salary of \$28,000 with a raise of \$3,000 each year. Company B offers a starting salary of \$36,000 with a raise of \$2,000 per year. Company C offers a starting salary of \$18,000. After how many years would the salaries for Companies A and B be the same? How much of a raise per year would Company C have to offer to equal the salaries of Companies A and B in the year in which the salaries of those two companies are the same?

Write a real-world situation that could be modeled by the equation. Then solve for the unknown in the situation.

10. \(60 + 25x = 35x + 20\)
LESSON 7-1
Equations with the Variable on Both Sides

Practice and Problem Solving: D

Use algebra tiles to model and solve each equation. The first one is done for you.
1. \(x + 2 = 2x - 1\)

\[
\begin{array}{c}
\text{tiles} \\
\text{models} \\
\text{simplified}
\end{array}
\]

\[x = 3\]

2. \(2x - 1 = x - 3\)

Fill in the boxes to solve each equation. The first one is done for you.
3. \(7y + 1 = 3y + 13\)

\[
\begin{array}{c}
-3y \\
4y + 1 = 13
\end{array}
\]

\[y = \frac{12}{4} = 3\]

4. \(4w + 3 = 2w + 7\)

\[
\begin{array}{c}
-2w \\
2w = 7
\end{array}
\]

\[w = \frac{7}{2}\]

5. \(-2r + 4 = -3r + 9\)

\[
\begin{array}{c}
+3r \\
r + 4 = 9
\end{array}
\]

\[r = 5\]

Solve. The first one is done for you.
6. \(2y + 1 = 3y - 5\)

\[
\begin{array}{c}
-2y \\
1 = y - 5
\end{array}
\]

\[y = 6\]

7. \(4x + 6 = -x + 1\)

8. \(-2y + 3 = 8y - 7\)

Write an equation to represent each relationship. Then solve the equation. The first one is done for you.
9. Four times a number minus 5 is the same as twice the number plus 3.

\[4n - 5 = 2n + 3; n = 4\]

10. Seven minus 2 times a number is the same as the number minus 2.
Reteach

If there are variable terms on both sides of an equation, first collect them on one side. Do this by adding or subtracting. When possible, collect the variables on the side of the equation where the coefficient will be positive.

Solve the equation $5x = 2x + 12$.

To collect on left side, subtract $2x$ from both sides of the equation.

Check: Substitute into the original equation.

$$5x = 2x + 12$$
$$5(4) \neq 2(4) + 12$$
$$20 \neq 8 + 12$$
$$20 = 20$$

Solve the equation $-6z + 28 = 9z - 2$.

To collect on right side, add $6z$ to both sides of the equation.

Check: Substitute into the original equation.

$$-6z + 28 = 9z - 2$$
$$-6(2) + 28 \neq 9(2) - 2$$
$$-12 + 28 \neq 18 - 2$$
$$16 = 16$$

Complete to solve and check each equation.

1. $9m + 2 = 3m - 10$

To collect on left side, subtract ___ from both sides.

Check: Substitute into the original equation.

$$9m + 2 = 3m - 10$$
$$9(\text{___}) + 2 \neq 3(\text{___}) - 10$$
$$\text{___} + 2 \neq \text{___} - 10$$
$$\text{___} = \text{___}$$

2. $-7d - 22 = 4d$

To collect on right side, add ___ to both sides.

Check: Substitute into the original equation.

$$-7d - 22 = 4d$$
$$-7(\text{___}) - 22 \neq 4(\text{___})$$
$$\text{___} - 22 \neq \text{___}$$
$$\text{___} = \text{___}$$
LESSON 7-1 
**Equations with the Variable on Both Sides**

*Reading Strategies: Follow a Procedure*

Equations may have variables on both sides. Follow these steps to get the variables on one side of the equation.

Solve $6x - 7 = 2x + 5$.

**Step 1:** Get all variables on one side of the equation.

\[
\begin{align*}
6x - 7 &= 2x + 5 \\
-2x &\quad -2x \\
4x - 7 &= 5
\end{align*}
\]

*Subtract $2x$ from both sides.*

**Step 2:** Get all constants on the other side of the equation.

\[
\begin{align*}
4x &= 12 \\
+7 &\quad +7
\end{align*}
\]

*Add 7 to both sides.*

**Step 3:** Solve.

\[
\begin{align*}
4x &= 12 \\
\frac{4}{4} &\quad \frac{12}{4} \\
x &= 3
\end{align*}
\]

*Divide both sides by 4.*

Use the above procedure to answer each question.

1. What is the first step to solve equations with variables on both sides?

2. What was done to get the variables on one side?

3. Write the equation with the variables on one side only.

4. What is the second step in solving the equation?

5. What was done to get the constants on one side?

6. What was the last step to solve the equation?
Problem 1

On Monday Elaine ran 5 miles and 3 laps around a trail. On Tuesday she ran 6 miles and 2 laps around a trail. She ran the same distance both days. How many miles long is one lap around the trail?

What information is given?

Monday's distance: 5 miles + 3 laps  Tuesday's distance: 6 miles + 2 laps

Monday's distance = Tuesday's distance

Write an equation: 5 + 3x = 6 + 2x
Solve the equation:

\[
\begin{align*}
5 + 3x &= 6 + 2x \\
-3x &= -2x \\
5 + x &= 6 \\
-5 &= -5 \\
x &= 1
\end{align*}
\]

One lap is 1 mile long.

1. Why should the length of the trail be the variable?

2. What do 2x and 3x represent?

3. To solve for the variable, what must all be together on one side of the equation?

4. Write a real-world situation that could be modeled by the equation 8 + 2x = 6x. Then solve the problem.

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Write the least common multiple of the denominators in the equation.

1. \( 9 + \frac{3}{4}x = \frac{7}{8}x - 10 \)  \hspace{1cm} 2. \( \frac{2}{3}x + \frac{1}{6} = -\frac{3}{4}x + 1 \)

Describe the operations used to solve the equation.

3. \( \frac{5}{6}x - 2 = -\frac{2}{3}x + 1 \)

\[
6\left( \frac{5}{6}x - 2 \right) = 6\left( -\frac{2}{3}x + 1 \right)
\]

\[
5x - 12 = -4x + 6
\]

\[
+4x \hspace{1cm} +4x
\]

\[
9x - 12 = 6
\]

\[
+12 \hspace{1cm} +12
\]

\[
9x = 18
\]

\[
\frac{9x}{9} = \frac{18}{9}
\]

\[
x = 2
\]

Solve.

4. \( \frac{2}{3}x + \frac{1}{3} = \frac{1}{3}x + \frac{2}{3} \)  \hspace{1cm} 5. \( \frac{3}{5}n + \frac{9}{10} = -\frac{1}{5}n - \frac{23}{10} \)  \hspace{1cm} 6. \( \frac{5}{6}h - \frac{7}{12} = \frac{3}{4}h - \frac{13}{6} \)

7. \( 4.5w = 5.1w - 30 \)  \hspace{1cm} 8. \( \frac{4}{7}y - 2 = \frac{3}{7}y + \frac{3}{14} \)  \hspace{1cm} 9. \( -0.8a - 8 = 0.2a \)

10. Write and solve a real-world problem that can be modeled by the equation \( 0.75x - 18.50 = 0.65x \).
LESSON 7-2  Equations with Rational Numbers

Practice and Problem Solving: C

Solve.

1. \(-\frac{2}{3}(x + 2) = \frac{1}{6}(x + 6)\)  
2. \(0.15 - 0.2x = 0.3(x + 3)\)  
3. \(\frac{2}{5}(r - 2) = \frac{3}{20}(r - 4)\)

4. \(\frac{1}{3}\left(x + \frac{2}{3}\right) = \frac{1}{6}(x - 4)\)  
5. \(0.4(0.3x - 1) = 2 + 0.75x\)  
6. \(-\frac{1}{2}(t + 9) + \frac{3}{4}t = \frac{3}{8}t\)

Answer each of the following.

7. If \(\frac{4}{3}x - \frac{2}{3} = -18\), what is the value of \(2x\)?

8. If \(0.9x - 1.3 = -3.1\), what is the value of \(x - 0.8\)?

9. Nikos sold muffins at his club’s bake sale. He spent $28.50 on supplies. He sold his muffins for $0.75 each, and made a profit of $36.75. Write and solve an equation to find out how many muffins Nikos sold.

Write a real-world situation that could be modeled by the equation. Then solve for the unknown in the situation.

10. \(3 + \frac{2}{3}x = \frac{5}{6}x - \frac{7}{8}\)
LESSON 7-2  Equations with Rational Numbers

Practice and Problem Solving: D

Write the least common multiple of the denominators in the equation. The first one is done for you.

1. \(6 + \frac{3}{4}x = \frac{1}{2}x - 4\)  
2. \(\frac{2}{3}x + \frac{1}{6} = -3x + 4\)

Describe the operations used to solve the equation. The first one is done for you.

3. \(\frac{7}{10}x - 2 = \frac{2}{5}x + 1\)

\[
10\left(\frac{7}{10}x - 2\right) = 10\left(\frac{2}{5}x + 1\right)
\]

\[
7x - 20 = 4x + 10
\]

\[
-4x - 4x
\]

\[
3x - 20 = 10
\]

\[
+20 + 20
\]

\[
3x = 30
\]

\[
\frac{3x}{3} = \frac{30}{3}
\]

\[
x = 10
\]

Solve. The first one is done for you.

4. \(\frac{7}{9} + n = 3n + \frac{1}{9}\)

5. \(\frac{1}{4} - \frac{1}{2}r = -\frac{3}{4}r\)

6. \(12.5 - 4g = -2g - 3.5\)

\(7 + 9n = 27n + 1\)

\(7 = 18n + 1\)

\(6 = 18n\)

\(n = \frac{1}{3}\)
LESSON 7-2 Equations with Rational Numbers

Reteach

To solve an equation with a variable on both sides that involves fractions, first get rid of the fractions.

Solve \( \frac{3}{4}m + 2 = \frac{2}{3}m + 5 \).

Multiply both sides of the equation by 12, the LCM of 4 and 3.

\[
12 \left( \frac{3}{4}m + 2 \right) = 12 \left( \frac{2}{3}m + 5 \right)
\]

Multiply each term by 12.

\[
12 \left( \frac{3}{4}m \right) + 12(2) = 12 \left( \frac{2}{3}m \right) + 12(5)
\]

Simplify.

\[
9m + 24 = 8m + 60
\]

Subtract 8m from both sides.

\[
9m - 8m + 24 = 8m - 8m + 60
\]

Simplify.

\[
m + 24 = 60
\]

Subtract 24 from both sides.

\[
m = 36
\]

Check: Substitute into the original equation.

\[
\frac{3}{4}m + 2 = \frac{2}{3}m + 5
\]

\[
\frac{3}{4}(36) + 2 = \frac{2}{3}(36) + 5
\]

\[
27 + 2 = 24 + 5
\]

\[
29 = 29
\]

Complete to solve and check your answer.

1. \( \frac{1}{4}x + 2 = \frac{2}{5}x - 1 \)

Multiply both sides of the equation by \( \frac{20}{1} \) the LCM of 4 and 5.

\[
\left[ \frac{20}{1} \right]\left( \frac{1}{4}x + 2 \right) = \left[ \frac{20}{1} \right]\left( \frac{2}{5}x - 1 \right)
\]

Multiply each term by \( \frac{20}{1} \).

\[
\left[ \frac{20}{1} \right]\left( \frac{1}{4}x \right) + \left[ \frac{20}{1} \right](2) = \left[ \frac{20}{1} \right]\left( \frac{2}{5}x \right) - \left[ \frac{20}{1} \right](1)
\]

Simplify.

\[
\frac{5}{1}x + \left[ \frac{40}{1} \right] = \left[ \frac{16}{1} \right]x - \left[ \frac{20}{1} \right]
\]

Subtract \( \frac{16}{1} \).

\[
\frac{5}{1}x - \frac{16}{1}x = \frac{40}{1} + \frac{20}{1}
\]

Simplify.

\[
\frac{9}{1}x = 60
\]

Add \( \frac{18}{1} \).

\[
\frac{9}{1}x = 60
\]

Divide both sides by \( \frac{9}{1} \).

\[
\frac{9}{1}x = \frac{60}{1}
\]

Simplify.

\[
x = \frac{20}{1}
\]
LESSON 7-2
Equations with Rational Numbers

Reading Strategies: Use a Sequence Chain

Use the sequence chain below to guide you in solving equations with rational numbers.

Are there fractions or decimals in the equation?
If fractions, multiply every term by the LCM.
If decimals, multiply every term by the same power of 10 to clear the decimals.

Are there variables on both sides of the = sign?
If yes, add or subtract to get them on one side.

Is a number being added or subtracted from the variable?
If yes, do the opposite to both sides.

Is the variable multiplied or divided by a number?
If yes, do the opposite to both sides.

Does your answer check using substitution?
If yes, you are done! If no, try again.

Answer each question.
1. When an equation contains fractions, what should you do before getting the variables together on one side?

2. When an equation contains decimals, what should you do before getting the variables together on one side?

Solve each equation using the sequence chain.
3. $\frac{3}{8}x - 4 = \frac{1}{8}x - 5$
4. $0.8 + 0.8k = 0.6k + 0.9$
5. $\frac{1}{3}y + 1 = \frac{1}{2}y - 3$
LESSON 7-2 Equations with Rational Numbers
Success for English Learners

Problem 1
Solve \(2 + \frac{2}{5}x = \frac{3}{8}x - 1\).

Eliminate the fractions.

\[
40 \left(2 + \frac{2}{5}x\right) = 40 \left(\frac{3}{8}x - 1\right)
\]

\[
40(2) + 4 \cdot 8 \left(\frac{2}{5}x\right) = 4 \cdot 5 \left(\frac{3}{8}x\right) - 40(1)
\]

\[
80 + 16x = 15x - 40
\]

The fractions are gone!

Problem 2
Solve \(0.4x = 0.375x + 1\).

Clear the decimals.

\[
1000(0.4x) = 1000(0.375x + 1)
\]

\[
400x = 375x + 1000
\]

The decimals are gone!

Solve.

1. Explain how you would solve the equation, \(80 + 16x = 15x - 40\), in Problem 1. Then solve for \(x\).

2. Write a real-world situation that could be modeled by the equation, \(0.4x = 0.375x + 1\), in Problem 2. Then solve for \(x\).

3. When an equation has fractions with denominators of 2, 3, and 4, what can you multiply the equation by to eliminate the fractions?

4. When an equation has decimals that are in tenths and hundredths, what can you multiply the equation by to clear the decimals?
LESSON 7-3  
Equations with the Distributive Property

Practice and Problem Solving: A/B

Solve each equation.

1. \(4(x - 2) = x + 10\)  
2. \(\frac{2}{3}(n - 6) = 5n - 43\)

3. \(-2(y + 12) = y - 9\)  
4. \(8(12 - k) = 3(k + 21)\)

5. \(8(-1 + m) + 3 = 2\left(m - \frac{5 + 1}{2}\right)\)  
6. \(2y - 3(2y - 3) + 2 = 31\)

Use the situation below to complete Exercises 7–8.

A taxi company charges $2.25 for the first mile and then $0.20 per mile for each additional mile, or \(F = 2.25 + 0.20(m - 1)\) where \(F\) is the fare and \(m\) is the number of miles.

7. If Juan’s taxi fare was $6.05, how many miles did he travel in the taxi?

8. If Juan’s taxi fare was $7.65, how many miles did he travel in the taxi?

Use the situation below to complete Exercises 9–11.

The equation used to estimate typing speed is \(S = \frac{1}{5}(w - 10e)\), where \(S\) is the accurate typing speed, \(w\) is the number of words typed in 5 minutes and \(e\) is the number of errors.

9. Ignacio can type 55 words per minute (wpm). In 5 minutes, she types 285 words. How many errors would you expect her to make?

10. If Alexis types 300 words in 5 minutes with 5 errors, what is his typing speed?

11. Johanna receives a report that says her typing speed is 65 words per minute. She knows that she made 4 errors in the 5-minute test. How many words did she type in 5 minutes?
Solve each equation.

1. \(2(x - 1) = x + 4\)
2. \(3(n - 6) = -5n - 2\)
3. \(-2(y - 5) = y + 1\)
4. \(8(8 - k) = -2(k - 5)\)
5. \(2\left(-4\frac{1}{2} + m\right) + 3 = 4(m - 3) + 5\frac{1}{2}\)
6. \(0.5(x - 12) + 2 = 1.25(x + 8) - 9.5\)

Write and solve an equation to find each solution.

7. One bag of trail mix has 5 ounces of raisins and some almonds. Lon buys 3 bags of trail mix and has 48 ounces of trail mix altogether. How many ounces of almonds are in each bag of trail mix?

8. A moving van charges a flat rate of $25 per day plus $0.12 per mile for every mile over 100 driven. If Millie’s bill was $29.46 how many miles to the nearest mile did she drive in all?

9. Benjamin is 4 years younger than Kevan. William is 4 years less than twice Benjamin’s age. If William is 22, how old are Kevan and Benjamin?

10. A taxi company charges $2 for the first mile and then $0.25 per mile for each additional mile. If Lupita’s fare was $4.50, how many miles did she travel in the taxi?

11. Parker has quarters and dimes in his piggy bank. He has 4 more dimes than quarters, and he has a total of $7.05 in his bank. How many dimes and quarters does Parker have?
LESSON 7-3  Equations with the Distributive Property

Practice and Problem Solving: D

Solve each equation. The first one has been done for you.

1. \(2(x - 5) = 10\)
   \[x = 10\]

2. \(3(n - 6) = 27\)

3. \(9(4 - s) = 10 + 4s\)

4. \(8(2 - p) = 24p\)

5. \(-2(y - 3) = y + 24\)

6. \(8(8 - k) = -72k\)

7. \(-3(12 - m) = -1(m - 8)\)

8. \(10(2 + x) = 15(x - 1) + 5\)

Answer each question to solve the problem. The first one is done for you.

9. Kevan is 6 years younger than his sister Katie. Melanie is twice as old as Kevan. How old are all three siblings?
   a. If you let \(k\) represent Katie’s current age, what expression can you use to represent Kevan’s current age?
      \[k - 6\]
   b. Based on your answer to part a, what expression represents Melanie’s age?
   c. If Melanie is 18 years old, what equation can you write to solve the problem?
   d. Solve the equation. How old are Kevan and Katie?
LESSON 7-3  Equations with the Distributive Property

**Reteach**

When solving an equation, it is important to simplify on both sides of the equal sign before you try to isolate the variable.

\[ 3(x + 4) + 2 = x + 10 \]

Since you cannot combine \( x \) and 4, multiply both by 3 using the Distributive Property.

\[ 3x + 12 + 2 = x + 10 \]

Then combine like terms.

\[ 3x + 14 = x + 10 \]

Subtract 14 to begin to isolate the variable term.

\[
\begin{align*}
3x & = x - 4 \\
-x & = -x \\
2x & = -4 \\
\frac{2x}{2} & = \frac{-4}{2} \\
x & = -2
\end{align*}
\]

The solution is \(-2\).

**Solve.**

1. \( 5(i + 2) - 9 = -17 - i \)

2. \( -3(n + 2) = n - 22 \)

You may need to distribute on both sides of the equal sign before simplifying.

\[ 3(3m - 2) = \frac{3}{4}(4 - 24m) \]

Use the Distributive Property on both sides of the equation to remove the parentheses.

\[
\begin{align*}
9m - 6 & = 3 - 18m \\
+6 & = +6 \\
9m & = 9 - 18m \\
+18m & = +18m \\
27m & = 9 \\
\frac{27}{27} & = \frac{9}{27} \\
m & = \frac{1}{3}
\end{align*}
\]

The solution is \( \frac{1}{3} \).

**Solve.**

3. \( 9(y - 4) = -10\left(y + 2 \frac{1}{3}\right) \)

4. \( -7\left(-6 - \frac{6}{7}x\right) = 12\left(x - 3 \frac{1}{2}\right) \)
LESSON 7-3
Equations with the Distributive Property

Reading Strategies: Follow a Procedure

Use the steps below to understand the procedure for solving equations using the distributive property.

Solve $2(x + 5) + 3 = 4(x + 2) - 11$.

\[
2(x + 5) + 3 = 4(x + 2) - 11
\]
\[
2x + 10 + 3 = 4x + 8 - 11
\]
\[
2x + 13 = 4x - 3
\]
\[
2x = 4x - 16
\]
\[
-4x
\]
\[
-2x = -16
\]
\[
-2
\]
\[
x = 8
\]

Solve each equation using the procedure shown. Show all your steps.

1. $-4(j + 2) - 3j = 6$

2. $4n + 6 - 2n = 3(n + 3) - 11$

3. $5(r - 1) = 2(r - 4) - 6$

4. $2 \left( n + \frac{1}{3} \right) = \frac{3}{2}n + 1$
Problem 1

Distribute on both sides.

\[ 3\left(q + \frac{1}{2}\right) = 4(q + 2) - 2\frac{1}{2} \]

Combine like terms.

\[ 3q + 1\frac{1}{2} = 4q + 8 - 2\frac{1}{2} \]
\[ 3q + 1\frac{1}{2} = 4q + 5\frac{1}{2} \]
\[ 3q = 4q + 4 \]
\[ -q = 4 \]
\[ q = -4 \]

Problem 2

Geri has exactly $3 in dimes and nickels. She has 9 more dimes than nickels.

How many of each coin does Geri have?

Let \( n \) = the number of nickels.

\[ 5\text{¢} \times n = \text{value of the nickels} \]

There are 9 more dimes than nickels.

The number of dimes = \( n + 9 \)

\[ 10\text{¢} \times (n + 9) = \text{value of the dimes} \]

Equation: value of the nickels + value of the dimes = $3

\[ 0.05n + 0.10(n + 9) = 3 \]
\[ 0.05n + 0.10n + 0.9 = 3 \]
\[ 0.15n + 0.9 = 3 \]
\[ 0.15n = 2.1 \]
\[ n = \frac{2.1}{0.15} = 14 \]

Geri has 14 nickels and 14 + 9 or 23 dimes.

Solve.

1. \(-1(x + 8) + 3 = x -15\)

2. Portia has 9 more pennies than quarters. Altogether she has $2.95 in pennies and quarters. How many of each coin does Portia have?
LESSON 7-4  Equations with Many Solutions or No Solution

Practice and Problem Solving: A/B

Tell whether each equation has one, zero, or infinitely many solutions. If the equation has one solution, solve the equation.

1. $4(x - 2) = 4x + 10$

2. $\frac{1}{2}n + 7 = \frac{n + 14}{2}$

3. $6(x - 1) = 6x - 1$

4. $6n + 7 - 2n - 14 = 5n + 1$

5. $4x + 5 = 9 + 4x$

6. $\frac{1}{2}(8 - x) = \frac{8 - x}{2}$

7. $8(y + 4) = 7y + 38$

8. $4(-8x + 12) = -26 - 32x$

9. $2(x + 12) = 3x + 24 - x$

10. $3x - 14 + 2(x - 9) = 2x - 2$

Solve.

11. Cell phone company A charges $20 per month plus $0.05 per text message. Cell phone company B charges $10 per month plus $0.07 per text message. Is there any number of text messages that will result in the exact same charge from both companies?

12. Lisa’s pet shop has 2 fish tanks. Tank A contains smaller fish who are fed 1 gram of food each per day. Tank B contains larger fish who are fed 2 grams of food each per day. If Tank B contains $\frac{2}{3}$ the number of fish that Tank A contains, will Lisa ever feed both tanks the same amount of food?
Tell whether each equation has one, zero, or infinitely many solutions. If the equation has one solution, solve the equation.

1. \[-(2x - 1) - 2x = 4(1 - x)\]

2. \[1 = \frac{-m - 3}{4} + \frac{m + 3}{5}\]

3. \[-3(2p + 1) + 4(2p - 3) = 2p - 15\]

4. \[6n + 7 - 2n - 14 = 5n + 1\]

5. \[\frac{4r + 2}{2} + 1 = r + 16\]

6. \[5(3 - m) + 10 = 5(5 - m)\]

7. \[16 + \frac{1}{4}(x + 8) = 9(x + 2)\]

8. \[-(q - 5) = \frac{8q - 4}{-4}\]

Solve.

9. Latrice and Meagan are collecting coins. Latrice doubled her coin collection the first month, and then added 200 coins to her collection the second month. Meagan added 85 coins to her collection the first week, and then doubled her total the second week. Now Latrice and Meagan have the same number of coins. Did they begin with the same number of coins? Explain.

10. Domingo picks up 5 coins from his collection of quarters and nickels. Is it possible for him to pick up exactly $0.50?
   
   a. Write an equation.

   b. Solve the equation.

   c. Does the equation have a solution?

   d. Does the word problem have a solution?
Equations with Many Solutions or No Solution

Practice and Problem Solving: D

Tell whether each equation has one, zero, or infinitely many solutions. The first one has been done for you. If the equation has one solution, solve the equation.

1. \(2x - 1 = 2x + 3\)
   
   \[\text{zero solutions}\]

2. \(3(y - 2) = 3y - 6\)

3. \(\frac{m - 3}{4} = \frac{m - 3}{5}\)

4. \(6n + 7 - 2n - 14 = 4n + 8\)

5. \(4r + 2 = r + 8\)

6. \(3m + 8 = 3(4 + m) - 4\)

7. \(\frac{1}{4}(x + 3) = 4(x + 3)\)

8. \(t - 2 = \frac{3t}{10} + 5\)

9. \(6(d - 4) = 18d\)

10. \(2(d + 7) - 1 = \frac{8d}{4} + 13\)

Answer each question. Begin with the equation \(x = x\). The first one has been done for you.

11. Add the same number to both sides of the equation \(x = x\).

   \[x + 5 = x + 5\]

12. Does your new equation have 0, 1 or many solutions?

13. Multiply both sides of your new equation by the same number. Be sure to use parentheses to group the sums before multiplying.

14. Does your new equation have 0, 1 or many solutions?

15. Use the Distributive Property on one side of your equation so that the two sides look different.

16. Does your new equation have 0, 1 or many solutions?
When you solve a linear equation, you are trying to find a value for the variable that makes the equation true. Often there is only one value that makes an equation true – one solution. But sometimes there is no value that will make the equation true. Other times there are many values that make the equation true.

\[
\begin{align*}
x + 3 &= 8 \\
x &= 5
\end{align*}
\]
Use properties of equality to solve.
If you get a statement that tells you what the variable equals, the equation has one solution.

\[
\begin{align*}
x + 3 &= x + 4 \\
3 &= 4
\end{align*}
\]
If you get a false statement with no variables, the equation has no solution.

\[
\begin{align*}
x + 3 &= x + 3 \\
3 &= 3
\end{align*}
\]
If you get a true statement with no variables, the equation has infinitely many solutions.

Tell whether each equation has one, zero, or infinitely many solutions.

1. \(5(i + 2) = 8(i - 1)\)

2. \(-3(n + 2) = -3n - 6\)

You can write an equation with one solution, no solution, or infinitely many solutions.

**One solution:** Start with a variable on one side and a constant on the other. This is your solution. Add, subtract, multiply or divide both sides of the equation by the same constant(s). Your equation has one solution. **Example:** \(3(r + 2) = 30\)

**No solution:** Start with a false statement of equality about two constants, such as \(3 = 4\). Now add, subtract, multiply or divide the same variable from both sides. You may then add, subtract, multiply or divide additional constants to both sides. Your equation has no solution. **Example:** \(k + 3 = k + 4\)

**Infinitely many solutions:** Start with a true statement of equality about two constants, such as \(5 = 5\). Now add, subtract, multiply or divide the same variable from both sides. You may then add, subtract, multiply or divide additional constants to both sides. Your equation has many solutions. **Example:** \(5(n - 3) = 5n - 15\)

Solve.

3. Write an equation with one solution. ____________________________

4. Write an equation with no solution. ____________________________

5. Write an equation with infinitely many solutions. ____________________________
LESSON 7-4
Equations with Many Solutions or No Solution
Reading Strategies: Compare and Contrast

Compare equations with zero, one, or many solutions.

<table>
<thead>
<tr>
<th>Equation</th>
<th>One Solution</th>
<th>Zero Solutions</th>
<th>Infinitely Many Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3(y + 2) = 30$</td>
<td>$5(2 + c) = 45 + 5c$</td>
<td>$2(v - 2) = 2v - 4$</td>
<td></td>
</tr>
<tr>
<td>Use properties of equality</td>
<td>$3y + 6 = 30$</td>
<td>$10 + 5c = 45 + 5c$</td>
<td>$2v - 4 = 2v - 4$</td>
</tr>
<tr>
<td>$3y = 24$</td>
<td>$10 = 45$</td>
<td>$-4 = -4$</td>
<td></td>
</tr>
<tr>
<td>$y = 8$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>End result</td>
<td>one solution</td>
<td>no solution</td>
<td>Infinitely many solutions</td>
</tr>
</tbody>
</table>

Tell whether each equation has 0, 1, or infinitely many solutions.
1. $2p + 8 = 2(p + 4)$
2. $4(t + 8) = 8t + 8 - 4t$

Compare how to write an equation with zero, one, or infinitely many solutions.

<table>
<thead>
<tr>
<th>Begin with</th>
<th>One Solution</th>
<th>Zero Solutions</th>
<th>Infinitely Many Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution: variable = constant</td>
<td>False statement: constant = constant</td>
<td>True statement: constant = constant</td>
<td></td>
</tr>
<tr>
<td>$y = 8$</td>
<td>$2 = 9$</td>
<td>$-2 = -2$</td>
<td></td>
</tr>
<tr>
<td>Make a change</td>
<td>$+, -, \times, \div$ by the same number on both sides</td>
<td>$+, -, \times, \div$ by the same variable on both sides</td>
<td>$+, -, \times, \div$ by the same variable on both sides</td>
</tr>
<tr>
<td>$y + 2 = 8 + 2$</td>
<td>$2 + c = 9 + c$</td>
<td>$v - 2 = v - 2$</td>
<td></td>
</tr>
<tr>
<td>$y + 2 = 10$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Make another change (optional)</td>
<td>$+, -, \times, \div$ by another number on both sides</td>
<td>$+, -, \times, \div$ by the same number on both sides</td>
<td>$+, -, \times, \div$ by the same number on both sides</td>
</tr>
<tr>
<td>$3(y + 2) = 30$</td>
<td>$5(2 + c) = 45 + 5c$</td>
<td>$2(v - 2) = 2v - 4$</td>
<td></td>
</tr>
</tbody>
</table>

Solve.
3. Write an equation with one solution. _______________________
4. Write an equation with no solution. _______________________
5. Write an equation with many solutions. _______________________
LESSON 7-4  Equations with Many Solutions or No Solution
Success for English Learners

Problem 1
Write an equation with no solution.

\[
4 = -2
\]
Start with a false statement.

\[
4y = -2y
\]
Add, subtract, multiply, or divide by the same variable on both sides.

\[
4y + 9 = -2y + 9
\]
Add, subtract, multiply, or divide by the same constant on both sides.

1. Write your own equation with no solution. Use at least two operations.

_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________

Problem 2
Write an equation with infinitely many solutions.

\[
9 = 9
\]
Start with a true statement.

\[
9 - r = 9 - r
\]
\(+, -, \times, \div\) by the same variable on both sides

\[
2(9 - r) = 18 - 2r
\]
\(+, -, \times, \div\) by the same constant on both sides

2. Write your own equation with infinitely many solutions. Use at least two operations.

_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________
Use the information to complete Exercises 1–7.

Happy Trails Ranch charges $25.00 for equipment rental plus $8.50 an hour to ride a horse. Rough Riders Ranch charges $20.75 for equipment rental plus $9.75 an hour to ride.

1. Which is the better deal if a customer plans on riding for 2.5 hours? Show your work.

_________________________________________________________________________________________
_________________________________________________________________________________________

2. When would the two ranches charge the same amount? Show your work.

_________________________________________________________________________________________
_________________________________________________________________________________________

3. Rewrite your equation from Exercise 2 using the Distributive Property and 5 as a factor. Solve.

_________________________________________________________________________________________
_________________________________________________________________________________________

4. How many solutions are there to $25.00 + 8.50x = 5(5 + 1.7x)$? Explain your answer.

_________________________________________________________________________________________
_________________________________________________________________________________________

5. Tina started riding at noon at Happy Trails and rode for 3.4 hours. At what time did she finish her ride? What did her ride cost?

_________________________________________________________________________________________

6. Pierre rode for 3.4 hours at Rough Riders. Without doing any computation can you tell how much his ride cost? Explain your reasoning.

_________________________________________________________________________________________

7. If a customer planned on riding for 5 hours, at which ranch would he or she get the better deal? Explain.

_________________________________________________________________________________________
Solve each linear system by graphing. Check your answer.

1. \( y = -1 \)
   \( y = 2x - 7 \)

2. \( x - y = 6 \)
   \( 2x = 12 + 2y \)

3. \( \frac{1}{2} x - y = 4 \)
   \( 2y = x + 6 \)

4. \( y = 4x - 3 \)
   \( 2y - 3x = 4 \)

5. Two skaters are racing toward the finish line of a race. The first skater has a 40 meter lead and is traveling at a rate of 12 meters per second. The second skater is traveling at a rate of 14 meters per second. How long will it take for the second skater to pass the first skater?
LESSON
8-1
Solving Systems of Linear Equations by Graphing

Practice and Problem Solving: C

Use the information below to complete Exercises 1–4.
Kelly needs to order lunch for orders 6 people at a business meeting. Her menu choices are chicken salad for a cost of $5 per person and egg salad for a cost of $4 per person. She only has $28 to spend. More people want chicken salad.

1. Write and graph one equation in a system for this situation. ____________________________

2. Write a second equation in the system. Graph it on the same grid. ________________________

3. What do \( x \) and \( y \) represent? ___________________________________________
   __________________________________________

4. How many of each type of lunch can she order? _______________________________________
   __________________________________________

Graph the lines for the two sets of linear data. Find the intersection of the lines.

5. \[ \begin{array}{c|c}
    x & y \\
    \hline
    -2 & 0 \\
    0 & 1 \\
    2 & 2 \\
    4 & 3 \\
\end{array} \]
   \[ \begin{array}{c|c}
    x & y \\
    \hline
    -2 & -1.5 \\
    0 & -3.5 \\
    2 & -5.5 \\
    4 & -7.5 \\
\end{array} \]

6. A softball team bought a box of sweatshirts for $240. Each sweatshirt cost $12 to print and will sell for $18. Graph a system of equations to find the number of sweatshirts the softball team needs to sell in order to break even. ________________________________________________________________
LESSON 8-1  Solving Systems of Linear Equations by Graphing

Practice and Problem Solving: D

Solve each linear system by graphing. Check your answer. The first one is done for you.

1. \( y = x + 3 \)
   \[ y = -2x + 6 \]

2. \( 3x = y \)
   \[ y = 2x + 2 \]

3. \( x = 3y \)
   \[ y = x - 4 \]

4. \( y = 5x - 4 \)
   \[ y - 5x = 1 \]

5. Wanda started walking along a path 27 seconds before Dave. Wanda walked at a constant rate of 3 feet per second. Dave walked along the same path at a constant rate of 4.5 feet per second. Graph the system of linear equations. How long after Dave starts walking will he catch up with Wanda?

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LESSON 8-1
Solving Systems of Linear Equations by Graphing
Reteach

When solving a system of linear equations by graphing, first write each equation in slope-intercept form. Do this by solving each equation for \( y \).

**Solve the following system of equations by graphing.**

\[
\begin{align*}
  y &= -2x + 3 \\
  y + 4x &= -1
\end{align*}
\]

The first equation is already solved for \( y \).

Write the second equation in slope-intercept form.

Solve for \( y \).

\[
\begin{align*}
  y + 4x - 4x &= -1 - 4x \\
  y &= -4x - 1
\end{align*}
\]

Graph both equations on the coordinate plane.

The lines intersect at \((-2, 7)\). This is the solution to the system of linear equations.

To check the answer, substitute \(-2\) for \( x \) and \( 7 \) for \( y \) in the original equations.

\[
\begin{align*}
  y &= -2x + 3; \quad 7 = -2(-2) + 3; \quad 7 = 4 + 3; \quad 7 = 7 \\
  y + 4x &= -1; \quad 7 + 4(-2) = -1; \quad 7 - 8 = -1; \quad -1 = -1
\end{align*}
\]

**Solve each linear system by graphing. Check your answer.**

1. \[
\begin{align*}
  y &= x + 1 \\
  y &= -x + 5
\end{align*}
\]

2. \[
\begin{align*}
  y + 3x &= 1 \\
  y - 6 &= 2x
\end{align*}
\]
A **system of linear equations** has two or more equations that are graphed on the same **coordinate grid**. The solution for the system is the ordered pair of the point where all of the equations intersect.

The solution to a system can come in three forms.

A system can have one solution. The system includes equations of different lines. The graph shows lines that intersect in one point.

1. Graph an example of a system of two equations that has one solution.

A system can have no solutions. The system includes equations of parallel lines. The graph shows parallel lines.

2. Graph an example of a system of two equations that has no solution.

A system can have infinitely many solutions. The system includes equations of the same line written in different forms. The graph shows a single line.

3. Graph an example of a system of two equations that has infinitely many solutions.

If you write all of the equations in a system in slope-intercept form, you can often tell how many solutions there are before you graph it. Look at the slopes.

If the slopes are different, there will be one solution.

If the slopes are the same, look at the $y$-intercept.

If the $y$-intercepts are different, there will be no solutions.

If the $y$-intercepts are the same, there will be infinitely many solutions.

**Without graphing, predict the number of solutions for each system of equations.**

4. $y = 2x + 5$
   $y = 2x - 1$

5. $y = \frac{3}{2}x$
   $y = 4x + 7$

6. $y = -x - 1$
   $y = -1 - x$

7. $y = 2x + 5$
   $y = -2x + 5$
Problem 1

Two Questions ➔ Two Equations ➔ One or Two Lines

Question 1: What is the distance, d for Plane 1?

Equation 1:
\[ d = 300t + 1200 \]

Equation 2:
\[ d = 500t \]

Question 2: What is the distance, d, for Plane 2?

The lines cross at (6, 3000).

1. How do you know that the system of equations shown in Problem 1 has only one solution?

_________________________________________________________________________________________

2. Explain how to check that the ordered pair in Problem 1 is the correct solution to the system of equations.

_________________________________________________________________________________________
_________________________________________________________________________________________

3. Explain how to graph the two linear equations in Problem 1.

_________________________________________________________________________________________

_________________________________________________________________________________________
LESSON 8-2 Solving Systems by Substitution

Practice and Problem Solving: A/B

Solve each system by substitution. Check your answer.

1. \[
\begin{align*}
y &= x - 2 \\
y &= 4x + 1
\end{align*}
\]

2. \[
\begin{align*}
2x - y &= 6 \\
x + y &= -3
\end{align*}
\]

3. \[
\begin{align*}
3x - 2y &= 7 \\
x + 3y &= -5
\end{align*}
\]

   (_____, _____)  (_____, _____)  (_____, _____)

Estimate the solution of each system by sketching its graph.

4. \[
\begin{align*}
y &= -4x + 5 \\
3x + 2y &= 0
\end{align*}
\]

5. \[
\begin{align*}
3x &= -y + 10 \\
2x + 3y &= -12
\end{align*}
\]

   Estimated solution:  Estimated solution:

   (about _____, about _____)  (about _____, about _____)

6. A sales associate in a department store earns a commission on each suit and each pair of shoes sold. One week, she earned $47 in commission for selling 3 suits and a pair of shoes. The next week, she earned $107 in commission for selling 7 suits and 2 pairs of shoes. How much commission does she earn for selling each suit and each pair of shoes? Solve by substitution.

   ____________________________________________________________________
LESSON 8-2
Solving Systems by Substitution

Practice and Problem Solving: C

Use the information below to complete Exercises 1–3.

The following three equations are shown on the graph:

A: \(x + y = 2\)
B: \(-3x + y = -2\)
C: \(y = 1\)

1. What is the solution of this system?
   \((\underline{\quad}, \underline{\quad})\)

2. How can you re-write Equation C so that it is satisfied by the system solution?

3. Two weavers are selling their products at a crafts fair. One weaver sells 4 blankets and 6 sweaters for $150. The other weaver sells 8 blankets and 12 sweaters for $400. Do the weavers charge the same amount for blankets and sweaters? Write and solve a system of equations to support your answer.

Re-write these systems to make it easier to estimate a solution.

4. \[
\begin{align*}
4.13x - \frac{23}{34}y &= 5.754 \\
-1.804x &= 10.04 + \frac{56}{11}y
\end{align*}
\]

5. \[
\begin{align*}
y &= 0.005 \\
0.006x + 0.00812y &= 0.00087
\end{align*}
\]

The linear equation \(195x - 221y = 65\) has an infinite number of solutions for \(x\) and \(y\). These are given by the linear equations \(x = 40 - 17n\) and \(y = 35 - 15n\), where \(n\) is any integer.

6. What is the smallest integer that will make \(x < 0\) and \(y < 0\)?

7. Is there any integer that will give the values \(x = 20\) and \(y = 30\)? Prove that your answer is correct.
LESSON 8-2  Solving Systems by Substitution

Practice and Problem Solving: D

Fill in the blanks to solve each system by substitution. The first one is started for you.

1. \[ \begin{align*}
   y &= 3x \\
   y &= x + 4
\end{align*} \]

Substitute \(3x\) for \(y\) in the second equation.

\[ 3x = x + 4 \]

\[ -x \quad -x \]

\[ ____ = 4 \]

\[ \div \quad \div \]

\[ ____ = ____ \]

Since \(x = ____\), substitute ____ for \(x\) in one of the equations to find the value of \(y\):

\[ y = 3x \]

\[ y = 3( ____ ) \]

\[ y = ____ \]

Solution: (____, ____)

2. \[ \begin{align*}
   3x + y &= 25 \\
   y &= x - 3
\end{align*} \]

Substitute _____ for \(y\) in the first equation.

\[ 3x + ( ____ ) = 25 \]

\[ ____ - 3 = 25 \]

\[ +3 \quad +3 \]

\[ ____ = 28 \]

\[ \div \quad \div \]

\[ ____ = ____ \]

Since \(x = ____\), substitute ____ for \(x\) in one of the equations to find the value of \(y\).

\[ y = x - 3 \]

\[ y = ____ - 3 \]

\[ y = ____ \]

Solution: (____, ____)

3. \[ \begin{align*}
   y &= 4x \\
   y &= 2x + 6
\end{align*} \]

4. \[ \begin{align*}
   y &= x - 2 \\
   2x + y &= 4
\end{align*} \]

Solve each system by substitution. Check your answer.

5. A decorator charges a $75 consultation fee, plus $50 per hour. Another decorator charges a $50 consultation fee, plus $60 per hour. When do the decorators change the same amount? How much is it? Solve.

1\textsuperscript{st} decorator: \(y = ____ x + ____\)

Write a system of equations for this problem.

1\textsuperscript{st} equation: ________________

2\textsuperscript{nd} equation: ________________

Solve the system of equations by substitution. \(x = ____\) hours; \(y = \$_{____}\)
Solving Systems by Substitution

Reteach

You can use substitution to solve a system of equations if one of the equations is already solved for a variable.

Solve \[
\begin{align*}
  y &= x + 2 \\
  3x + y &= 10
\end{align*}
\]

Step 1: Choose the equation to use as the substitute.
Use the first equation \( y = x + 2 \) because it is already solved for a variable.

Step 2: Solve by substitution.
\[
\begin{align*}
  y &= x + 2 \\
  3x + y &= 10
\end{align*}
\]

\[
\begin{align*}
  3x + (x + 2) &= 10 \\
  4x + 2 &= 10 \\
  4x &= 8 \\
  x &= 2
\end{align*}
\]

Step 3: Now substitute \( x = 2 \) back into one of the original equations to find the value of \( y \).
\[
\begin{align*}
  y &= x + 2 \\
  y &= 2 + 2 \\
  y &= 4
\end{align*}
\]

The solution is \((2, 4)\).

Check:
Substitute \((2, 4)\) into both equations.
\[
\begin{align*}
  y &= x + 2 \\
  3x + y &= 10 \\
  4 \overset{?}{=} 2 + 2 \\
  3(2) + 4 \overset{?}{=} 10 \\
  4 \overset{?}{=} 4 \checkmark \\
  6 + 4 \overset{?}{=} 10 \\
  10 \overset{?}{=} 10 \checkmark
\end{align*}
\]

Solve each system by substitution. Check your answer.

1. \[
\begin{align*}
  x &= y - 1 \\
  x + 2y &= 8
\end{align*}
\]

2. \[
\begin{align*}
  y &= x + 2 \\
  y &= 2x - 5
\end{align*}
\]

3. \[
\begin{align*}
  y &= x + 5 \\
  3x + y &= -11
\end{align*}
\]

4. \[
\begin{align*}
  x &= y + 10 \\
  x &= 2y + 3
\end{align*}
\]
Solving Systems by Substitution

**Reading Strategies: Build Vocabulary**

When you solve and check a system of equations by substitution, you may see words such as “in the place of,” or “replace.”

**Example**

Solve the system by substitution.  
\[ \begin{align*} 
4x - y &= 8 \\
-2x + 3y &= -6 
\end{align*} \]

**Solution**

The term substitution means to **replace** one of the variables, \( x \) or \( y \), in one of the equations with its value from the **other** equation. Look at the equations and decide which variable would be the easiest to find.

Here, it’s the \( y \) in \( 4x - y = 8 \). Add a “+y” to both sides of the equation, and subtract 8 from both sides of the equation to give \( 4x - 8 = y \).

Then, substitute this value for \( y \) in the other equation and simplify:

\[ 
\begin{align*} 
-2x + 3y &= -6 \\
\rightarrow -2x + 3(4x - 8) &= -6 \\
\rightarrow -2x + 12x - 24 &= -6, \; \text{or} \; 10x = 18 
\end{align*} \]

This gives \( x = \frac{9}{5} \). Next, replace \( x \) with \( \frac{9}{5} \) in **either** of the equations in the system. For the first equation, this gives \( 4 \left( \frac{9}{5} \right) - y = 8 \), which simplifies to \( 36 - 5y = 40 \), or \( y = -\frac{4}{5} \). So, the solution of the system is \( \left( \frac{9}{5}, -\frac{4}{5} \right) \).

**Check**

You might graph the equations on a coordinate plane to check your answer. If you round the numbers in the solutions, about where will you find the point of intersection?

The \( x \) value of \( \frac{9}{5} \) is **about** 2. The \( y \) value of \( -\frac{4}{5} \) is **about** \(-1\). So, the point of intersection would be at **about** \((2, -1)\).

**Solve each system by substitution.**

1. \[ \begin{align*} 
2x &= 3y \\
y &= x - 2 
\end{align*} \]

   Solution: (______, ____)

2. \[ \begin{align*} 
x + y &= 2 \\
-x &= 2y - 7 
\end{align*} \]

   Solution: (______, ____)

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LESSON 8-2
Solving Systems by Substitution
Success for English Learners

Problem 1

Equation 1
\[ 2x + y = 5 \]

Equation 2
\[ y = x - 4 \]

Step 1: Substitute

Step 2: Solve

\[ 2x + (x - 4) = 5 \]
\[ 2x + x - 4 = 5 \]
\[ 3x = 9 \]
\[ x = 3 \]

Solution: \((3, -1)\)

Step 3: Re-substitute

Graph to Check:

Problem 2

Option #1

This will cost $50.

Let \( m \) = number of months.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( m = 1 )</th>
<th>( m = 2 )</th>
<th>( m = 3 )</th>
<th>( m = 4 )</th>
<th>( m = 5 )</th>
<th>( m = 6 )</th>
<th>( m = 7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>$50</td>
<td>$80</td>
<td>$110</td>
<td>$140</td>
<td>$170</td>
<td>$200</td>
<td>$230</td>
</tr>
<tr>
<td>(Monthly fee) ( \cdot m )</td>
<td>$30</td>
<td>$60</td>
<td>$90</td>
<td>$120</td>
<td>$150</td>
<td>$180</td>
<td>$210</td>
</tr>
</tbody>
</table>

Option #2

This is FREE!

Let \( m \) = number of months.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( m = 1 )</th>
<th>( m = 2 )</th>
<th>( m = 3 )</th>
<th>( m = 4 )</th>
<th>( m = 5 )</th>
<th>( m = 6 )</th>
<th>( m = 7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>$0</td>
<td>$40</td>
<td>$80</td>
<td>$120</td>
<td>$160</td>
<td>$200</td>
<td>$240</td>
</tr>
<tr>
<td>(Monthly fee) ( \cdot m )</td>
<td>$40</td>
<td>$80</td>
<td>$120</td>
<td>$160</td>
<td>$200</td>
<td>$240</td>
<td>$280</td>
</tr>
</tbody>
</table>

1. Once you find the value of \( x \) in Problem 1, what do you need to do?

2. Describe the two differences between Options 1 and 2 in Problem 2.
Solve each system by eliminating one of the variables by addition or subtraction. Check your answer.

1. \[
\begin{align*}
   x - y &= 8 \\
   x + y &= 12
\end{align*}
\]

2. \[
\begin{align*}
   2x - y &= 4 \\
   3x + y &= 6
\end{align*}
\]

3. \[
\begin{align*}
   x + 2y &= 10 \\
   x + 4y &= 14
\end{align*}
\]

4. \[
\begin{align*}
   3x + y &= 9 \\
   y &= 3x + 6
\end{align*}
\]

5. \[
\begin{align*}
   4x + 5y &= 15 \\
   6x - 5y &= 18
\end{align*}
\]

6. \[
\begin{align*}
   5x &= 7y \\
   x + 7y &= 21
\end{align*}
\]

Write a system of equations for each problem. Solve the system using elimination. Show your work and check your answers.

7. Aaron bought a bagel and 3 muffins for $7.25. Bea bought a bagel and 2 muffins for $6. How much is a bagel and how much is a muffin?

8. Two movie tickets and 3 snacks are $24. Three movie tickets and 4 snacks are $35. How much is a movie ticket and how much is a snack?

Explain why the system has the answer given. Solve each system by elimination to prove your answer.

9. \[
\begin{align*}
   x + 2y &= 8 \\
   x + 2y &= 20
\end{align*}
\]

10. \[
\begin{align*}
   3x + y &= 9 \\
   3x &= 9 - y
\end{align*}
\]

No solution

Infinitely many solutions
Solving Systems by Elimination

Practice and Problem Solving: C

Answer the questions below.

1. Solve the system by elimination:
   \[ x - y = 4 \]  Equation A
   \[ x + z = 6 \]  Equation B
   \[ y - z = 8 \]  Equation C

   a. \[ x + y = ____ \]  Equation D
   
   b. Add the new Equation D to Equation A to eliminate the variable y.

   ______________________

   c. \[ x = ____ \]

   Substitute \( x = 9 \) into Equation A to find \( y \) and into Equation B to find \( z \).

   \[ x - y = 4 \rightarrow 9 - y = 4, \text{ or } y = 5 \]

   d. \[ x + z = 6 \rightarrow 9 + z = 6, \text{ or } z = ____ \]

   e. The solution is \( x = \____, y = \____, \text{ and } z = \____ \), which can be written as an ordered triple \( (______, ______, ______) \).

Solve by elimination.

2. \[ 2x = y + 4 \]  \[ 2a + 3b = 5 \]
   \[ y = 2z + 5 \]  \[ a + 3c = 6 \]
   \[ 2z = x + 3 \]  \[ 3b + c = 7 \]

   \( (______, ______, ______) \)  \( (______, ______, ______) \)

3. \[ 2b - c = 5 \]  \[ 2x + y = 1 \]
   \[ a + c = 10 \]  \[ y + z = -2 \]
   \[ 2b - a = 1 \]  \[ 3x - 2z = 4.5 \]

   \( (______, ______, ______) \)  \( (______, ______, ______) \)
Solving Systems by Elimination

Practice and Problem Solving: D

Solve the systems by elimination. The first one is started for you.

1. \[ \begin{align*}
3x + 3y &= 14 \\
2x - 3y &= -8 
\end{align*} \]

Add the equations:

\[ x + 3y = 14 \]
\[ +2x - 3y = -8 \]

\[ 3x + 0 = 6 \]

or

\[ \frac{3x}{3} = 6 \]
\[ x = 2 \]

Substitute \( x = 2 \) for \( x \) in one of the equations:

\[ x + 3y = 14 \]
\[ \frac{3y}{3} = \frac{14 - 2}{3} \]
\[ y = \frac{12}{3} \]
\[ y = 4 \]

Solution: \((2, 4)\)

2. \[ \begin{align*}
2x + 2y &= 4 \\
x + 2y &= 7 
\end{align*} \]

Subtract the equations:

\[ 2x + 2y = 4 \]
\[ - (3x + 2y = 7) \]

\[ -x + 0 = -3 \]

\[ \frac{-x}{-1} = \frac{-3}{-1} \]
\[ x = 3 \]

Substitute \( x = 3 \) for \( x \) in one of the equations:

\[ x + 2y = 7 \]
\[ \frac{2y}{2} = \frac{7 - 3}{2} \]
\[ y = \frac{4}{2} \]
\[ y = 2 \]

Solution: \((3, 2)\)

3. \[ \begin{align*}
3x + 4y &= 26 \\
x - 2y &= -8 
\end{align*} \]

Multiply the second equation by 2. Then, add the equations:

\[ 3x + 4y = 26 \]
\[ 2(x - 2y = -8) \]

\[ \frac{3x}{3} + \frac{4y}{2} = 26 + \frac{2(-8)}{2} \]
\[ x + 2y = -2 \]

\[ \frac{x}{1} = \frac{-2}{1} \]
\[ x = -2 \]

Substitute \( x = -2 \) for \( x \) in one of the equations:

\[ 3x + 4y = 26 \]
\[ 3(-2) + 4y = 26 \]
\[ -6 + 4y = 26 \]
\[ 4y = 32 \]
\[ \frac{4y}{4} = \frac{32}{4} \]
\[ y = 8 \]

Solution: \((-2, 8)\)

Solve each system by elimination.

4. \[ \begin{align*}
3x - 2y &= 1 \\
2x + 2y &= 14 
\end{align*} \]

5. \[ \begin{align*}
x + y &= 4 \\
3x + y &= 16 
\end{align*} \]

6. \[ \begin{align*}
3x + 2y &= -26 \\
2x - 6y &= -10 
\end{align*} \]
Solving a system of two equations in two unknowns by elimination can be done by adding or subtracting one equation from the other.

**Elimination by Adding**
Solve the system: \( x + 4y = 8 \)
\( 3x - 4y = 8 \)

\[ \text{Solution} \]
Notice that the terms \("+4y"\) and \("-4y"\) are opposites. This means that the two equations can be added without changing the signs.

\[
\begin{align*}
  x + 4y &= 8 \\
  3x - 4y &= 8 \\
  4x + 0 &= 16 \\
  4x &= 16, \text{ or } x = 4
\end{align*}
\]

Substitute \( x = 4 \) in either of the equations to find \( y \):
\[
\begin{align*}
  x + 4y &= 8 \\
  4y &= 4, \text{ or } y = 1
\end{align*}
\]

The solution of this system is \((4, 1)\).

**Elimination by Subtracting**
Solve the system: \( 2x - 5y = 15 \)
\( 2x + 3y = -9 \)

\[ \text{Solution} \]
Notice that the terms \("2x"\) are common to both equations. However, to eliminate them, it is necessary to subtract one equation from the other. This means that the signs of one equation will change. Here, the top equation stays the same. The signs of the bottom equation change.

\[
\begin{align*}
  2x - 5y &= 15 \\
  (-)2x (-)3y &= (+)9 \\
  0 - 8y &= 24, \text{ or } y = -3
\end{align*}
\]

Substitute \( y = -3 \) in either of the original equations to find \( x \):
\[
\begin{align*}
  2x - 5y &= 15 \\
  2x - 5(-3) &= 15 \\
  2x + 15 &= 15, \text{ or } x = 0
\end{align*}
\]

The solution of this system is \((0, -3)\).

Solve the following systems by elimination. State whether addition or subtraction is used to eliminate one of the variables.

1. \[
\begin{align*}
  3x + 2y &= 10 \\
  3x - 2y &= 14
\end{align*}
\]
   Operation: __________________
   Solution: (______, _____)

2. \[
\begin{align*}
  x + y &= 12 \\
  2x + y &= 6
\end{align*}
\]
   Operation: __________________
   Solution: (______, _____)
LESSON 8-3  Solving Systems by Elimination

Reading Strategies: Analyze Information

When solving word problems using systems, identify the two variables. This is especially important if you are trying to find a variable with the same coefficient that can be eliminated by addition or subtraction.

Example
A welcome kit contains 3 grocery store coupons and 7 drug store coupons. The coupons are worth $45. Another welcome kit contains 3 grocery store coupons and 6 drug store coupons, worth $42. What is the value of each grocery coupon and each drug store coupon?

Solution
To use the elimination method, it is important to notice that both welcome kits contain 3 grocery store coupons. This means that the grocery-store coupon variable can be eliminated by subtracting one equation from the other.

The equation for the first welcome kit is $3g + 7d = 45$.
The equation for the second welcome kit is $3g + 6d = 42$. Write one equation over the other and subtract the like terms:

$$
\begin{align*}
3g + 7d &= 45 \\
(-)3g (-)6d &= (-)42
\end{align*}
$$

$$1d = 3$$

The drug store coupons have a value of $3.

Substitute to find the average value of the grocery coupon:

$3g + 7(3) = 45$, $3g = 24$, and $g = 8$

The grocery store coupons have a value of $8.

Identify the variable that is the same in each equation in the system that is used to solve the problem. Then, add or subtract to eliminate that variable. Finally, solve the system.

1. Joy bought 2 bath towels from a linen store, but returned 3 hand towels. Kay bought 3 bath towels and 3 hand towels. Joy’s bill was $5. Kay’s bill was $45. What are the prices of the bath and hand towels?

2. One family spent $45 on movie tickets for 2 adults and 3 children. Another family spent $40 for 2 adults and 2 children. What are the prices of the adult movie tickets and the child movie tickets?
LESSON 8-3
Solving Systems by Elimination
Success for English Learners

Problem 1

STEP 1:

\[
\begin{align*}
\begin{cases}
x - 2y &= -19 \\
5x + 2y &= 1
\end{cases}
\end{align*}
\]

STEP 2:

\[
\begin{align*}
x - 2y &= -19 \\
5x + 2y &= 1 \\
6x &= -18 \\
x &= -3
\end{align*}
\]

STEP 3:

\[
\begin{align*}
x - 2y &= -19 \\
-3 - 2y &= -19 \\
+3 &+ 3 \\
-2y &= -16 \\
y &= 8
\end{align*}
\]

STEP 4:
Write the solution: \((-3, 8)\)

Problem 2

A family buys 3 lunch combos and 2 desserts for $24. Another family buys 5 lunch combos and 2 desserts for $40. Write a system of equations for the dinners and desserts bought by the families.

Let \(L\) stand for the number of lunch combos. Let \(D\) stand for the number of desserts.

1. In Problem 1, what happens to the \(y\)-variable in Step 2 when the equations are added?

2. In Problem 2, what do you get if you subtract the top equation from the bottom equation? How much is a lunch combo?
LESSON 8-4  Solving Systems by Elimination with Multiplication

Practice and Problem Solving: A/B

Name the least common multiple (LCM) of the coefficients of each pair of variables. Ignore the signs.

1. \[
\begin{align*}
\begin{cases}
-2a + 3b &= 9 \\
4a - 2b &= 3
\end{cases}
\end{align*}
\]

LCM for a: _________  
LCM for b: _________

2. \[
\begin{align*}
\begin{cases}
7x + 5y &= 20 \\
6x - 18y &= 11
\end{cases}
\end{align*}
\]

LCM for x: _________  
LCM for y: _________

3. \[
\begin{align*}
\begin{cases}
-8m - 5n &= 3 \\
3m - 12n &= -1
\end{cases}
\end{align*}
\]

LCM for m: _________  
LCM for n: _________

Solve each system by elimination, multiplication, and addition or subtraction. Show and check your work.

4. \[
\begin{align*}
\begin{cases}
x + 3y &= -14 \\
2x - 4y &= 30
\end{cases}
\end{align*}
\]

(______, ____ )

5. \[
\begin{align*}
\begin{cases}
4x - y &= -5 \\
-2x + 3y &= 10
\end{cases}
\end{align*}
\]

(______, ____ )

6. \[
\begin{align*}
\begin{cases}
y - 3x &= 11 \\
2y - x &= 2
\end{cases}
\end{align*}
\]

(______, ____ )

Write and solve a system of linear equations for each problem. Show and check your work.

7. One family spends $134 on 2 adult tickets and 3 youth tickets at an amusement park. Another family spends $146 on 3 adult tickets and 2 youth tickets at the same park. What is the price of a youth ticket?

_________________________________________________________________________________________
_________________________________________________________________________________________

8. A baker buys 19 apples of two different varieties to make pies. The total cost of the apples is $5.10. Granny Smith apples cost $0.25 each and Gala apples cost $0.30. How many of each type of apple did the baker buy?

_________________________________________________________________________________________
_________________________________________________________________________________________
Solving Systems by Elimination with Multiplication

Practice and Problem Solving: C

Find the solution to each system.

1. \[
\begin{align*}
2x - 3y & = 4 \\
3x - 4y & = 5
\end{align*}
\]
2. \[
\begin{align*}
3x - 4y & = 5 \\
4x - 5y & = 6
\end{align*}
\]
3. \[
\begin{align*}
4x - 5y & = 6 \\
5x - 6y & = 7
\end{align*}
\]

(______, ____), (______, ____), (______, ____)

4. What do you notice about the coefficients of the variables and the constants on each system?

5. What do you predict the solution for the system \(5x - 6y = 7\) and \(6x - 7y = 8\) will be? Explain your answer.

6. Write the equations in Exercise 1 in slope-intercept form. How do the slopes compare?

7. What is happening to the slopes of the lines in Exercises 1–3? Toward what number are the slopes approaching?

8. What would the graph of the six equations in Exercises 1–3 look like?

Find the solution to the system in Exercise 9. Then, write two systems for Exercise 10 and 11 that have the same solution. Verify your equations by solving those systems.

9. \[
\begin{align*}
2x + 3y & = 4 \\
3x + 4y & = 5
\end{align*}
\]
10. __________________
11. __________________

(______, ____), (______, ____), (______, ____)

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LESSON 8-4 Solving Systems by Elimination with Multiplication

Practice and Problem Solving: D

Fill in the blanks to solve each system by elimination with multiplication. The first one has been started for you.

1. \[
\begin{align*}
3x + 4y &= 26 \\
x - 2y &= -8 \\
\end{align*}
\]

Multiply the second equation by 2. Then, add the equations:

\[
\begin{align*}
&3x + 4y = 26 \\
&2(x - 2y = -8) \\
&3x + 4y = 26 \\
&2x - 4y = -16 \\
&5x = 0 \\
&x = \frac{0}{5} \\
\end{align*}
\]

Substitute _____ for x in one of the equations:

\[
\begin{align*}
x - 2y &= -8 \\
-2y &= -8 \\
2y &= 8 \\
y &= 4 \\
\end{align*}
\]

Solution: (______, ______)

2. Last month, Stephanie spent $57 on 4 allergy shots and 1 office visit. This month, she spent $9 after 1 office visit and a refund for 2 allergy shots from her insurance company. How much do an allergy shot and an office visit cost?

Complete the equations:

\[
\begin{align*}
4a + 1v &= \rule{1cm}{.1pt} \\
-2a + 1v &= \rule{1cm}{.1pt} \\
\end{align*}
\]

Allergy shot: $_______; office visit: $_______
Elimination is used to solve a system of equations by adding like terms. Sometimes, it is necessary to multiply one or both equations by a number to use this method. You should examine the equations carefully and choose the coefficients that are easiest to eliminate.

**Multiplying one equation by a number**

\[
\begin{align*}
2x + 5y &= 9 \\
x - 3y &= 10
\end{align*}
\]

The easiest variable to work with is \(x\). Multiply the second equation by \(-2\).

\[
\begin{align*}
2x + 5y &= 9 \\
-2(x - 3y) &= -2(10)
\end{align*}
\]

\[
\begin{align*}
2x + 5y &= 9 \\
-2x + 6y &= -20 \\
0 + 11y &= -11
\end{align*}
\]

So, \(y = -1\), and \(2x + 5(-1) = 9\), or \(x = 7\). The solution is \((7, -1)\).

**Multiplying both equations by a number**

\[
\begin{align*}
5x + 3y &= 2 \\
4x + 2y &= 10
\end{align*}
\]

The least common multiple (LCM) of 4 and 5 is 20, and the LCM of 2 and 3 is 6. So, multiply the first equation by \(-2\) and the second equation by 3.

\[
\begin{align*}
-2(5x + 3y) &= -2(2) \\
3(4x + 2y) &= 3(10)
\end{align*}
\]

\[
\begin{align*}
-10x - 6y &= -4 \\
12x + 6y &= 30
\end{align*}
\]

\[
2x + 0 = 26
\]

So, \(x = 13\), and \(5(13) + 3y = 2\), so \(y = -21\). The solution is \((13, -21)\).

Solve each system by elimination.

1. \[
\begin{align*}
2x - y &= 20 \\
3x + 2y &= 19
\end{align*}
\]

\((______, _____)\)

2. \[
\begin{align*}
-3a + 4b &= 15 \\
5a - 6b &= 12
\end{align*}
\]

\((______, _____)\)

3. \[
\begin{align*}
3m + 5n &= 20 \\
4m - 6n &= 30
\end{align*}
\]

\((______, _____)\)

4. \[
\begin{align*}
3u - v &= 20 \\
-4u - 2v &= 13
\end{align*}
\]

\((______, _____)\)
LESSON 8-4 Solving Systems by Elimination with Multiplication

Reading Strategies: Connect Concepts

When solving systems of linear equations using elimination, you will sometimes need to multiply one or both equations by a factor in order to get the same coefficients for a variable. This process is very similar to getting a common denominator for fractions. Look at the example below.

\[
\begin{align*}
6x - 5y &= 16 \\
4x - 3y &= 12
\end{align*}
\]

To eliminate the \(x\)-terms, you need to get the same or opposite coefficients for \(x\) in both equations.

\[
\begin{align*}
Add: \quad &\frac{5}{6} + \frac{1}{4} = \frac{12}{12} \quad \text{(Equation 1)} \\
4 \cdot : &\quad 4,8,12,16,20 \\
6 \cdot : &\quad 6,12,18,24,30
\end{align*}
\]

Think about finding a common denominator for 4 and 6.

\[
\begin{align*}
2(6x - 5y &= 16) \\
3(4x - 3y &= 12)
\end{align*}
\]

Find the least common multiple (LCM) of 4 and 6 by listing their multiples in order. The LCM is 12.

\[
\begin{align*}
12x - 10y &= 32 \\
12x - 9y &= 36
\end{align*}
\]

Determine what you have to multiply 4 and 6 by to get 12. Multiply each equation by the appropriate number.

\[
\begin{align*}
12x - 10y &= 32 \\
12x - 9y &= 36
\end{align*}
\]

Now either add or subtract the equations. In this case, you will subtract the equations.

1. Describe how you would get common \(y\)-coefficients (instead of \(x\)-) in the example above.

_________________________________________________________________________________________

_________________________________________________________________________________________

2. Show how to get a set of common \(y\)-coefficients for the system \[
\begin{align*}
9x - 10y &= 7 \\
5x + 8y &= 31
\end{align*}
\]

_________________________________________________________________________________________

Solve each system of equations by elimination.

3. \[
\begin{align*}
9x - 2y &= 15 \\
4x + 3y &= -5
\end{align*}
\]

4. \[
\begin{align*}
2x - 3y &= 50 \\
7x + 8y &= -10
\end{align*}
\]

\((_____ , _____)\) \quad \((_____ , _____)\)
Problem 1
Solve the system by elimination and use multiplication.

\[
\begin{align*}
2x + y &= 3 \\
-x + 3y &= -12 \\
x + 4y &= -9
\end{align*}
\]

Neither variable is eliminated.

Multiply the 2nd equation by 2.

\[
\begin{align*}
2x + y &= 3 \\
2(-x + 3y) &= 2(-12) \\
2x + 6y &= -24
\end{align*}
\]

\[
\begin{align*}
7y &= -21
\end{align*}
\]

Solution: (3, -3)

Problem 2
How do you eliminate a variable in a system like this?

\[
\begin{align*}
2a + 3b &= 4 \\
3a - 4b &= 8
\end{align*}
\]

\[
\begin{align*}
4(2a + 3b) &= 4(4) \\
3(3a - 4b) &= 8(3)
\end{align*}
\]

\[
\begin{align*}
8a + 12b &= 16 \\
9a - 12b &= 24
\end{align*}
\]

\[
\begin{align*}
17a &= 40 \\
a &= \frac{40}{17} = \frac{2}{12}
\end{align*}
\]

Notice that \(3b\) and \(-4b\) have opposite signs. What is the smallest number that 3 and 4 divide evenly? 12

Multiply the 1st equation by 4. Multiply the 2nd equation by 3. The \(b\) is eliminated.

Solution: \(\left[\frac{2}{17}, \frac{6}{17}, -\frac{4}{17}\right]\)

1. In Problem 1, what would you multiply the first equation by to eliminate the \(y\) variable?

2. In Problem 2, what would you multiply the equations by to eliminate the variable \(a\)? (Hint: There are two different numbers, one for each equation.)
Graph each system. Describe the solution.

1. \[ \begin{align*}
    y &= x + 2 \\
    x - y &= 2
\end{align*} \]

2. \[ \begin{align*}
    x + 2y &= 5 \\
    3x &= 15 - 6y
\end{align*} \]

Solution: __________________________   ________________________

Re-write each system in the form \( y = mx + b \). Then, state whether the system has one solution, no solution, or many solutions without actually solving the system.

3. \[ \begin{align*}
    2x + y &= 1 \\
    2x + y &= -3
\end{align*} \]

4. \[ \begin{align*}
    y - 2 &= -5x \\
    y - 5x &= 2
\end{align*} \]

5. \[ \begin{align*}
    y - 3x + 2 &= 0 \\
    2 &= -y + 3x
\end{align*} \]

Solve.

6. Two sisters open savings accounts with $60 each that their grandmother gave them. The first sister adds $20 each month to her account. The second sister adds $40 every two months to her $60. If the sisters continue to make deposits at the same rate, when will they have the same amount of money?
Solving Special Systems

Practice and Problem Solving: C

Answer the questions about the graph.

1. Give the slope of the lines.
   - Slope of A: ___________
   - Slope of B: ___________
   - Slope of C: ___________

2. Write the equation of each line.
   - Line A: _______________________
   - Line B: _______________________
   - Line C: _______________________

3. What is the relationship of the slopes of lines A and B?
   _______________________________________________________________________

4. What is the relationship of the slopes of lines B and C?
   _______________________________________________________________________

5. Solve the system of equations represented by lines A and B.
   _______________________________________________________________________

6. Describe how you would use the distance formula,
   \[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \]
   and the Pythagorean Theorem to show that lines A and B are perpendicular.
   _______________________________________________________________________

Which lines are perpendicular? Write “yes” or “no.” Explain.

7. \[ \begin{align*}
    x + 3y &= 6 \\
    3x - y &= 9
\end{align*} \]
   _______________________________________________________________________

8. \[ \begin{align*}
    -2x + y &= 10 \\
    2x - 4y &= 15
\end{align*} \]
   _______________________________________________________________________

9. \[ \begin{align*}
    3x - 5y &= 21 \\
    5x + 3y &= 25
\end{align*} \]
   _______________________________________________________________________
Solving Special Systems

Answer the questions about the system. The first one is done for you.

\[
\begin{align*}
2x + 6y &= 15 \\
-3x - 9y &= 5
\end{align*}
\]

Write each equation in slope-intercept form, \( y = mx + b \).

1. **Step 1** Subtract the \( x \) term from both sides of each equation.
   
   \[
   \begin{align*}
   2x + 6y &= 15 \\
   -3x - 9y &= 5
   \end{align*}
   \]
   
   \[
   \begin{align*}
   2x - 2x + 6y &= 15 - 2x \\
   -3x + \underline{\_\_\_} - 9y &= 5 + \underline{\_\_\_}x
   \end{align*}
   \]
   
   \[
   \begin{align*}
   6y &= -2x + 15 \\
   -9y &= \underline{\_\_\_}x + 5
   \end{align*}
   \]
   
   \[
   \begin{align*}
   \frac{6y}{6} &= \frac{-2x}{6} + \frac{15}{6} \\
   \frac{-9y}{-9} &= \underline{\_\_\_}x + \frac{5}{-9}
   \end{align*}
   \]
   
   \[
   \begin{align*}
   y &= \frac{-1x + 5}{3} \\
   y &= \underline{\_\_\_}x - \frac{5}{2}
   \end{align*}
   \]

2. **Step 2** Write the slope, \( m \), of each equation.
   
   Slope of 1st equation: \( -\frac{1}{3} \)
   
   Slope of 2nd equation: _________

3. **Step 3** Compare the \( y \)-intercept, \( b \), of each equation.
   
   \[
   \begin{align*}
   y\text{-intercept, 1st equation: } \frac{5}{2} \\
   y\text{-intercept, 2nd equation: } \underline{\_\_\_}\n   \end{align*}
   \]

4. Based on your answers to Exercises 2 and 3, how many solutions does this system have? Explain.

________________________________________________________________________________________

How many solutions do these systems have: none, many, or one?

The first one is done for you.

5. \[
\begin{align*}
2y + 5x &= 8 \\
5x + 2y &= 17
\end{align*}
\]

6. \[
\begin{align*}
3x - 5y &= 40 \\
3x - 5y &= 80
\end{align*}
\]

7. \[
\begin{align*}
x - y &= 100 \\
10x - 10y &= 1,000
\end{align*}
\]

none
When solving equations in one variable, it is possible to have one solution, no solutions, or infinitely many solutions. The same results can occur when graphing systems of equations.

Solve \( \begin{align*}
4x + 2y &= 2 \\
2x + y &= 4
\end{align*} \)

Multiplying the second equation by \(-2\) will eliminate the \(x\)-terms.

\[
\begin{align*}
4x + 2y &= 2 \\
-2(2x + y) &= -4 \\
-4x - 2y &= -8 \\
0 + 0 &= -6 \\
0 &= -6
\end{align*}
\]

The equation is false. There is no solution.

Solve \( \begin{align*}
y &= 4 - 3x \\
3x + y &= 4
\end{align*} \)

Because the first equation is solved for a variable, use substitution.

\[
\begin{align*}
3x + (4 - 3x) &= 4 \\
0 + 4 &= 4 \\
0 &= -6
\end{align*}
\]

The equation is true for all values of \(x\) and \(y\). There are infinitely many solutions.

Graphing the system shows that these are parallel lines. They will never intersect, so there is no solution.

Solve each system of linear equations algebraically.

1. \( \begin{align*}
y &= 3x \\
2y &= 6x
\end{align*} \)

2. \( \begin{align*}
y &= 2x + 5 \\
y - 2x &= 1
\end{align*} \)

3. \( \begin{align*}
3x - 2y &= 9 \\
-6x + 4y &= 1
\end{align*} \)
A table can help you answer questions about special systems of equations.

<table>
<thead>
<tr>
<th>Number of Solutions</th>
<th>Similarities and Differences in $y = mx + b$</th>
<th>Description of Graphed Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>different slopes ($m$)</td>
<td>intersecting</td>
</tr>
<tr>
<td>infinitely many</td>
<td>same slope ($m$)</td>
<td>same lines</td>
</tr>
<tr>
<td></td>
<td>same $y$-int. ($b$)</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>same slope ($m$)</td>
<td>parallel</td>
</tr>
<tr>
<td></td>
<td>different $y$-int. ($b$)</td>
<td></td>
</tr>
</tbody>
</table>

**Answer the questions.**

1. A student solved a system by elimination as shown. How many solutions does it have? Explain.

   \[
   \begin{align*}
   2x + 3y &= 5 \\
   2x + 3y &= 5 \\
   2x + 3y &= 7 \\
   -(2x + 3y &= 7) \\
   0 &= -2
   \end{align*}
   \]

2. The graph of a system consists of two intersecting lines. How many solutions does the system have?
Problem 1

\[
\begin{align*}
\begin{cases}
y = x - 1 \\
x - y = 2
\end{cases} & \quad \begin{cases}
y + x = -1 \\
x - y = -2
\end{cases} & \quad \begin{cases}
y + x = -1 \\
2x - y = 0
\end{cases}
\end{align*}
\]

FALSE. There is no solution.

\[
\begin{align*}
\begin{cases}
y = 2x + 1 \\
2x - y + 1 = 0
\end{cases} & \quad \begin{cases}
y - 2x = 1 \\
2x - y = -1
\end{cases} & \quad \begin{cases}
y - 2x = 1 \\
2x - y = 0
\end{cases}
\end{align*}
\]

TRUE. There are infinitely many solutions.

Problem 2

<table>
<thead>
<tr>
<th>Number of Solutions</th>
<th>1</th>
<th>$\infty$ = Infinitely many</th>
<th>$\emptyset$ = No Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description of Equation</td>
<td>Different slopes: $m = -1$ and $m = 3$</td>
<td>Same slopes: $m = -1$ and $m = -1$</td>
<td>Same slopes: $m = -1$ and $m = -1$</td>
</tr>
<tr>
<td></td>
<td>$b = 2$ and $b = 2$</td>
<td>$b = 2$ and $b = 2$</td>
<td>Different $y$-intercepts: $b = 2$ and $b = -2$</td>
</tr>
<tr>
<td>Description of Graph</td>
<td>Intersecting lines</td>
<td>Same line</td>
<td>Parallel lines</td>
</tr>
<tr>
<td>Examples of Graphs</td>
<td><img src="image1" alt="Graph 1" /></td>
<td><img src="image2" alt="Graph 2" /></td>
<td><img src="image3" alt="Graph 3" /></td>
</tr>
</tbody>
</table>

1. In Problem 1, two different outcomes occur when the equations are added together. What are those outcomes?

2. How do the outcomes in Exercise 1 relate to the number of solutions of the system?
Solving Systems of Linear Equations

Challenge

In a village, 200 bushels of corn are distributed among 100 persons. Each child under the age of 6 gets one bushel of corn, each person age 60 or older gets two bushels of corn, and every other person gets three bushels of corn.

1. Let the variables $c$, $e$, and $p$ stand for each child, older person, and other person, respectively. Write a linear equation that relates the variables to the total number of persons in the village.

2. Write a linear equation for the way the 200 bushels of corn is distributed among the 100 persons using the variables $c$, $e$, and $p$.

3. Write a system of equations using your answers to Exercises 1 and 2.

4. Subtract the equation from Exercise 1 from the equation for Exercise 2. What do you get?

A third equation is needed to completely solve this problem, because there are three variables.

5. Use your answer to Exercise 4 to give three values for the variables in the new equation.

6. Check the values in Exercise 5 against the equations in Exercises 1 and 2 to make sure that a reasonable value for the third variable results. Then, write three possible values for all three of the variables.

$c$: _________ bushels $e$: _________ bushels $p$: _________ bushels

$c$: _________ bushels $e$: _________ bushels $p$: _________ bushels

$c$: _________ bushels $e$: _________ bushels $p$: _________ bushels
UNIT 3: Solving Equations and Systems of Equations

MODULE 7 Solving Linear Equations

LESSON 7-1

Practice and Problem Solving: A/B

1. \( x = -4 \)
2. \( x = 4 \)
3. \( x = 1 \)
4. \( 4a - 3 = 2a + 7 \)
   \[ \begin{align*}
   -2a & \quad \begin{array}{c}
   \begin{array}{c}
   2a - 3 = 7 \\
   +[3] + 3
   \end{array}
   \end{array} \\
   2a & = [10] \\
   a & = [5]
   \end{align*} \]
5. \( 7x - 1 = 2x + 5 \)
   \[ \begin{align*}
   -2x & \quad \begin{array}{c}
   \begin{array}{c}
   5x - 1 = [5] \\
   +[1] + 1
   \end{array}
   \end{array} \\
   5x & = [6] \\
   x & = [\frac{6}{5}]
   \end{align*} \]
6. \( -3r + 9 = -4r + 5 \)
   \[ \begin{align*}
   +4r & \quad \begin{array}{c}
   \begin{array}{c}
   r + 9 = 5 \\
   -[9] - 9
   \end{array}
   \end{array} \\
   r & = [-4]
   \end{align*} \]
7. \( y = 7 \)

Practice and Problem Solving: C

1. \( v = \frac{1}{2} \)
2. \( x = 3 \)
3. \( r = -3 \)
4. \( m = -4 \)
5. \( x = -1 \)
6. \( t = -5 \)
7. \( 12 - 2n = 8(n + 4); n = -2 \)
8. \( 3 + 8n = 3(2n - 1); n = -3 \)
9. \( 28,000 + 3,000n = 36,000 + 2,000n; \)
   \( n = 8; \) Company C would need to give a raise of \$4,250 per year to equal the salaries of Companies A and B, \$52,000, after year 8.
10. Sample answer: Erin has already saved \$60. She plans to save an additional \$25 per week. Robin has already saved \$20 and plans to save an additional \$35 per week. After how many weeks will Robin and Erin have the same amount saved?
    \( x = 4 \) weeks

Practice and Problem Solving: D

1. \( x = 3 \)
2. \( x = -2 \)
3. \[7y + 1 = 3y + 13\]
\[-[3y] - 3y\]
\[4y + 1 = 13\]
\[-1 - [1]\]
\[4y = [12]\]
\[4y = 12\]
\[y = [3]\]

4. \[4w + 3 = 2w + 7\]
\[-[2w] - 2w\]
\[2w + 3 = 7\]
\[-3 - [3]\]
\[2w = [4]\]
\[2w = 4\]
\[w = [2]\]

5. \[-2r + 4 = -3r + 9\]
\[+[3r] + 3r\]
\[r + 4 = 9\]
\[-[4] - 4\]
\[r = [5]\]

6. \[y = 6\]

7. \[x = -1\]

8. \[y = 1\]

9. \[4n - 5 = 2n + 3; n = 4\]

10. \[7 - 2n = n - 2; n = 3\]

---

**Reteach**

1. \[9m + 2 = 3m - 10\]
\[-[3m] - [3m]\]
\[6m + 2 = -10\]
\[-[2] - [2]\]
\[6m = [-12]\]
\[6m = -12\]
\[[6] = [6]\]
\[m = [-2]\]

To collect on left side, subtract \(3m\) from both sides.

Subtract 2 from both sides.

Divide by 6.

**Check**: Substitute into the original equation.

\[9m + 2 = 3m - 10\]

\[9(-2) + 2 \neq 3(-2) - 10\]

\[-18 + 2 \neq -6 - 10\]

\[-16 = -16\]

2. \[-7d - 22 = 4d\]
\[+[7d] + [7d]\]
\[-22 = 11d\]
\[-22 = 11d\]
\[[11] = [11]\]
\[[-2] = d\]

To collect on right side, add \(7d\) to both sides.

Divide by 11.

**Check**: Substitute into the original equation.

\[-7d - 22 = 4d\]

\[-7(-2) - 22 \neq 4(-2)\]

\[14 - 22 \neq -8\]

\[-8 = -8\]
Reading Strategies
1. Get all the variables on one side of the equation.
2. \(2x\) was subtracted from both sides.
3. \(4x - 7 = 5\)
4. Get all the constants on the other side of the equation.
5. 7 was added to both sides of the equation.
6. Both sides of the equation were divided by 4.

Success for English Learners
1. The length of the trail is the unknown value being solved for.
2. \(2x\): distance of 2 laps around a trail in miles; \(3x\): distance of 3 laps around a trail in miles.
3. The variables must be all on one side and the constants must all be on the other side.
4. Sample answer: On Monday Julie ran 8 miles and two laps around a trail. On Tuesday she ran 6 laps around the trail. She ran the same distance both days. How many miles long is one lap around the trail?
   \(x = 2\); One lap is 2 miles long.

LESSON 7-2

Practice and Problem Solving: A/B
1. 8
2. 12
3.
\[
6 \left( \frac{5}{6} x - 2 \right) = 6 \left( \frac{-2}{3} x + 1 \right)
\]
Multiply both sides by the LCM, 6.
\[
5x - 12 = -4x + 6
\]
Simplify.
\[
+4x \quad -4x
\]
Add 4x to both sides.
\[
9x - 12 = 6
\]
Simplify.
\[
+12 \quad +12
\]
Add 12 to both sides.
\[
9x = 18
\]
Simplify.
\[
9x = 18
\]
Divide both sides by 9.
4. \(x = 1\)
5. \(n = -4\)
6. \(h = -1\)
7. \(w = 50\)
8. \(y = 15 \frac{1}{2}\)
9. \(a = -8\)
10. Tina sold bags of popcorn at a bake sale. In the morning, Tina paid the booth fee of $18.50 and sold the bags for $0.75 each. In the afternoon she sold the bags for $0.65 each. Her profit in the morning was the same as her profit in the afternoon. How many bags of popcorn did Tina sell in the morning?
   \(x = 185\); Tina sold 185 bags in the morning.

Practice and Problem Solving: C
1. \(x = -2 \frac{4}{5}\)
2. \(x = -1.5\)
3. \(r = \frac{4}{5}\)
4. \(x = -5 \frac{1}{3}\)
5. \(x = -3 \frac{17}{21}\)
6. \(t = \frac{12}{13}\)
7. \(2x = -\frac{23}{3} \text{ or } -7 \frac{2}{3}\)
8. \(x - 0.8 = -2.8\)
9. \(0.75x - 28.50 = 36.75\); \(x = 87\) muffins
10. Possible answer: Three more than two-thirds the number of hours Laura worked last week is the same as five-sixths times the hours she worked this week decreased by seven-eighths.
    \(x = 23 \frac{1}{4}\) hours

Practice and Problem Solving: D
1. 4
2. 6
3.

\[
10 \left( \frac{7}{10} x - 2 \right) = 10 \left( \frac{2}{5} x + 1 \right)
\]

Multiply both sides by the LCM (10, 5), which is 10.

\[
7x - 20 = 4x + 10
\]

Simplify.

\[
-4x
\]

Subtract 4x from both sides.

\[
3x - 20 = 10
\]

Simplify.

\[
+20
\]

Add 20 to both sides.

\[
3x = 30
\]

Simplify.

\[
\frac{3x}{3} = \frac{30}{3}
\]

Divide both sides by 3.

\[
x = 10
\]

4. \( n = \frac{1}{3} \)

5. \( r = -1 \)

6. \( g = 8 \)

Reteach

1.

\[
[20]\left( \frac{1}{4} x + 2 \right) = [20] \left( \frac{2}{5} x - 1 \right)
\]

\[
[20] \left( \frac{4x}{5} \right) + [20](2) = [20] \left( \frac{2x}{5} \right) - [20](1)
\]

\[
\]

\[
-5x
\]

Subtract 5x.

\[
40 = 3x - 20
\]

\[
+20
\]

Add 20.

\[
[60] = 3x
\]

\[
60 \quad 3x
\]

\[
[3] = [3]
\]

\[
[20] = x
\]

Multiply both sides of the equation by 20 the LCM of 4 and 5.

Multiply each term by 20.

Simplify.

Subtract 5x.

Simplify.

Add 20.

Simplify.

Divide both sides by 3.

Simplify.

Check: Substitute into the original equation.

\[
\frac{1}{4} x + 2 = \frac{2}{5} x - 1
\]

\[
\frac{1}{4} (20) + 2 = \frac{2}{5} (20) - 1
\]

\[
5 + 2 \neq 8 - 1
\]

\[
7 = 7
\]

Reading Strategies

1. Multiply every term by the LCM.

2. Multiply every term by a power of 10 to clear the decimals.

3. \( x = -4 \)

4. \( k = \frac{1}{2} \)

5. \( y = 24 \)

Success for English Learners

1. Subtract 15x from both sides. Then subtract 80 from both sides. \( x = -120 \)

2. Sample answer: After adding 1 pound of peanuts to a bag that is 0.375 full the bag is now 0.4 full. How many pounds of peanuts does the bag hold when the bag is full? \( x = 40 \) pounds

3. LCM (2, 3, 4) = 12

4. 100

LESSON 7-3

Practice and Problem Solving: A/B

1. \( x = 6 \)

2. \( n = 9 \)

3. \( y = -5 \)

4. \( k = 3 \)

5. \( m = -1 \)

6. \( y = -5 \)

7. \( 20 \) mi

8. 28 mi

9. 1 error

10. 50 wpm

11. 365 words
Practice and Problem Solving: C
1. \( x = 6 \)
2. \( n = 2 \)
3. \( y = 3 \)
4. \( k = 9 \)
5. \( m = \frac{1}{4} \)
6. \( x = -6 \)
7. 11 oz
8. 137 mi
9. Benjamin: 13; Kevan: 17
10. 11 mi
11. 19 quarters, 23 dimes

Practice and Problem Solving: D
1. \( x = 10 \)
2. \( n = 15 \)
3. \( s = 2 \)
4. \( p = \frac{1}{2} \)
5. \( y = -6 \)
6. \( k = -1 \)
7. \( m = 11 \)
8. \( x = 6 \)
9. a. \( k = 6 \)
   b. \( 2(k - 6) \)
   c. \( 2(k - 6) = 18 \)
   d. Kevan is 9 and Katie is 15.

Reteach
1. \( i = -3 \)
2. \( n = 4 \)
3. \( y = \frac{2}{3} \)
4. \( x = 14 \)

Reading Strategies
1. \(-4(j + z) - 3j = 6\)
   \(-4j - 8 - 3j = 6\)
   \(-7j - 8 = 6\)
   \(-7j = 1\)
   \(j = -2\)

2. \(4n + 6 - 2n = 3(n + 3) - 11\)
   \(4n + 6 - 2n = 3n + 9 - 11\)
   \(2n + 6 = 3n - 2\)
   \(8 = n\)
3. \(5(r - 1) = 2(r - 4) - 6\)
   \(5r - 5 = 2r - 8 - 6\)
   \(5r - 5 = 2r - 14\)
   \(3r = -9\)
   \(r = -3\)
4. \(2\left(n + \frac{1}{3}\right) = \frac{3}{2}n + 1\)
   \(2n + \frac{2}{3} = \frac{3}{2}n + 1\)
   \(n = \frac{1}{3}\)
   \(n = \frac{2}{3}\)

Success for English Learners
1. \( x = 5 \)
2. 11 quarters; 20 pennies

LESSON 7-4
Practice and Problem Solving: A/B
1. zero solutions
2. infinitely many solutions
3. zero solutions
4. \( n = -8 \); one solution
5. zero solutions
6. infinitely many solutions
7. \( y = 6 \); one solution
8. zero solutions
9. infinitely many solutions
10. \( x = 10 \); one solution
11. Yes; 500 text messages will cost exactly the same from both companies.
12. No, the two tanks will never need the exact same amount of food.

Practice and Problem Solving: C
1. zero solutions
2. one solution; \( m = 7 \)
3. infinitely many solutions
4. one solution; \( n = -8 \)
5. one solution; \( r = 14 \)
6. infinitely many solutions
7. one solution; \( x = 0 \)
8. one solution; \( q = -4 \)
9. No; \( x + 2x + 200 = x + 85 + 2x \) results in no solutions.
10. a. Possible equation: \( 0.25x + 0.05(5 - x) = 0.50 \)
    
    b. \( x = 1 \frac{1}{4} \)
    
    c. yes
    
    d. no; it is not possible to have \( 1 \frac{1}{4} \) coins.

Practice and Problem Solving: D
1. zero solutions
2. infinitely many solutions
3. zero solutions
4. zero solutions
5. one solution; \( r = 2 \)
6. infinitely many solutions
7. one solution; \( x = -3 \)
8. one solution; \( t = 10 \)
9. one solution; \( d = -2 \)
10. infinitely many solutions
11. Any number may be added; Sample answer: \( x + 2 = x + 2 \)
12. infinitely many solutions
13. Any number may be used to multiply; Sample answer: \( 4(x + 2) = 4(x + 2) \)
14. infinitely many solutions
15. sample: \( 4x + 20 = 4(x + 5) \)
16. infinitely many solutions

Reteach
1. \( i = 6 \); one solution
2. infinitely many solutions
3. Answers may vary; should have one solution
4. Answers may vary; should have no solution
5. Answers may vary; should have infinitely many solutions

Reading Strategies
1. infinitely many solutions

2. no solution
3. Answers may vary; should have one solution
4. Answers may vary; should have no solution
5. Answers may vary; should have infinitely many solutions

Success for English Learners
1. Answers will vary; should have no solution
2. Answers will vary; should have infinitely many solutions

Module 7 Challenge
1. HT: \( 25.00 + 8.50 \times 2.5 \) = 46.25; RR: \( 20.75 + 9.75 \times 2.5 \) = 45.13; Rough Riders Ranch
2. \( 25.00 + 8.50x = 20.75 + 9.75x; 4.25 = 1.25x; 3.4 = x \); The two ranches would charge the same amount for 3.4 hours.
3. \( 5(5 + 1.7x) = 5(4.15 + 1.95x); 0.85 = 0.25x; 3.4 = x \)
4. infinitely many, the two sides of the equation are the same so any value of \( x \) will satisfy both
5. at 3:24 P.M.; $53.90
6. $53.90; For 3.4 hours, the two ranches charge the same amount.
7. Happy Trails; for any time over 3.4 hours Happy Trails is the better deal

MODULE 8 Solving Systems of Linear Equations

LESSON 8-1

Practice and Problem Solving: A/B
1. (3, -1)
2. \( \text{infinitely many solutions} \)

3. \( \text{no solution} \)

4. \( (2, 5) \)

5. \( 20 \text{ seconds} \)

---

**Practice and Problem Solving: C**

1. \( x + y = 6 \)

2. \( 5x + 4y = 28 \)

3. \( x \) represents the number of chicken salads and \( y \) represents the number of egg salads

4. \( (4, 2); 4 \text{ chicken salads and 2 egg salads} \)

5. \( (-3, -0.5) \)

6. \( 40 \text{ sweatshirts} \)
Practice and Problem Solving: D

1. 

2. 

3. 

4. 

5. 

81 seconds

Reteach

1. 

2. 

Reading Strategies

1. Drawings will vary. Sample drawing:
2. Drawings will vary. Sample drawing:

3. Drawings will vary. Sample drawing:

4. none
5. one
6. infinite
7. one

Success for English Learners
1. The two lines intersect at one point, so there is only one solution.
2. Substitute the ordered pair into each of the two equations, and check that both equations are true.
3. Sample answer: I plotted the y-intercept, and then used the slope to find another point. I then connected the two points with a straight line.

LESSON 8-2

Practice and Problem Solving: A/B
1. (−1, −3)
2. (1, −4)
3. (1, −2)

Practice and Problem Solving: C
1. 1; 1
2. 0 • x + y = 1
3. No. The equations 4B + 6S = 150 and 8B + 12S = 400 have no solution, which means there are no values of B and S that satisfy both equations.
4. Round the coefficients.
   \[ 4x - \frac{2}{3}y = 6 \text{ and } -2x = 10 + 6y \]
5. Multiply the coefficients by 1,000 and round: y = 5; 6x + 8y = 1

(2, −3)

(6, −8)

6. $13/suit, $8/pair of shoes
6. Solve the inequalities $40 - 17n < 0$ and $35 - 15n < 0$ to find the smallest integers that make $x < 0$ and $y < 0$. $n$ has to be 3 or greater.

7. Substitute $x = 20$ and $y = 30$ in the equations to see if an integer results; it does not in either case.

**Practice and Problem Solving: D**

1. $2x; 2; 2; x; 2; 2; 6; 2; 6$
2. $x - 3; x - 3; 4x; 4x; 4; 4; x; 7; 7; 7; 4; 7; 4$
3. $(3, 12)$
4. $(2, 0)$
5. $50; 75; 60; 50; y = 50x + 75; y = 60x + 50; 2.5; 200$

For 2.5 hours both decorators charge the same amount, $200.

**Reteach**

1. $(2, 3)$
2. $(7, 9)$
3. $(-4, 1)$
4. $(17, 7)$

**Reading Strategies**

1. $(6, 4)$
2. $(-3, 5)$

**Success for English Learners**

1. Substitute the value of $x$ into one of the equations to find $y$.
2. Option 1 charges $50 to set up the service and then $30 each month. Option 2 charges nothing to set up the service, but charges $40 each month.

**LESSON 8-3**

**Practice and Problem Solving: A/B**

1. $(10, 2)$
2. $(2, 0)$
3. $(6, 2)$
4. $\left(\frac{1}{2}, \frac{15}{2}\right)$
5. $\left(\frac{33}{10}, \frac{18}{50}\right)$

6. $\left(\frac{7}{2}, \frac{5}{2}\right)$

7. $b + 3m = 7.25; b + 2m = 6.00;\$
   $\$3.50/bagel, $1.25/muffin
8. $2m + 3s = 25; 3m + 4s = 35;\$
   $\$9.00/ticket, $2.00/snack

9. Answers may vary, but students should realize that when the equations are subtracted, an untrue statement results ($0 = -12$), which means that there is no common solution. A graph of this system will show two parallel lines.

10. Answers may vary, but students should realize that when the equations are subtracted, a true statement results ($0 = 0$), which means that there are many combinations of $x$ and $y$ that make the equations true statements. A graph of this system will show only one line, since both equations have the same graph.

**Practice and Problem Solving: C**

1. a. 14
2. $2x = 18$, or $x = 9$
3. 9
4. $-3$
5. $x = 9, y = 5, and z = -3; (9, 5, -3)$
6. $(12, 20, \frac{15}{2})$.
7. $(0, \frac{5}{3}, 2)$.
8. $(7, 4, 3)$
9. $(1.5, -2, 0)$

**Practice and Problem Solving: D**

1. $0, 3x, 3, 3, 2, 2, 2, 2, 12, 4; 2, 4$
2. $2y; -7; 0y; -3; -3; -1; -1; 3; 3; 3; 9; 9; -2; 2; 2; -1; 3; -1$
3. $2; 4; -16; 5; 10; 5; 10; 5; 2; 2; 2; 2; -10; -2; -2; 5; 2; 5$
4. $(3, 4)$
5. $(6, -2)$
6. $(-8, -1)$
Reteach
1. Addition; (4, –1)
2. Subtraction; (–6, 18)

Reading Strategies
1. The hand towels are the variable to eliminate. Bath towels: $10; hand towels: $5
2. The adult movie ticket is the variable to eliminate. Adult tickets: $15; child tickets: $5

Success for English Learners
1. The $y$-variable is eliminated. $−2y + 2y = 0$
2. The “2D” in both equations is eliminated, and you get $2L = 16$. The lunch combo is $8$.

LESSON 8-4

Practice and Problem Solving: A/B
1. 4; 6
2. 42; 90
3. 24; 60
4. \( \begin{pmatrix} 17 \\ 5 \end{pmatrix}, \begin{pmatrix} -29 \\ 5 \end{pmatrix} \)
5. \( \begin{pmatrix} -1 \\ 2 \end{pmatrix}, 3 \)
6. (–4, –1)
7. $2a + 3y = 134; 3a + 2y = 146; $22
8. $x + y = 19; 0.25x + 0.3y = 5.1; 7$ Galas, 12 Granny Smith’s

Practice and Problem Solving: C
1. (–1, –2)
2. (–1, –2)
3. (–1, –2)
4. The numbers are consecutive whole numbers.
5. (–1, –2). They follow the pattern.
6. $y = \frac{2}{3}x - \frac{4}{3}; y = \frac{3}{4}x - \frac{5}{4};$ both slopes are close to but less than 1.
7. The slopes are getting larger and approaching positive one.
8. All of the lines would intersect at (–1, –2) but would have different slopes less than +1 and different y-intercepts.
9. (–1, 2)

10–11. Answers may vary, but the two systems should have equations of the form $nx + (n + 1)y = n + 2$ and $(n + 3)x + (n + 4) y = (n + 5)$, where $n$ is an integer; (–1, 2), (–1, 2).

Practice and Problem Solving: D
1. 2; 4; –16; 5; 10; 5; 10; 5; 5; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2
2. 57; 9; 8; 25

Reteach
1. \( \begin{pmatrix} \frac{59}{7} \\ -\frac{22}{7} \end{pmatrix} \)
2. \( \begin{pmatrix} 69 \\ \frac{111}{2} \end{pmatrix} \)
3. \( \begin{pmatrix} \frac{135}{19} \\ -\frac{5}{19} \end{pmatrix} \)
4. \( \begin{pmatrix} \frac{27}{10} \\ -\frac{119}{10} \end{pmatrix} \)

Reading Strategies
1. Multiply the first equation by 3 and the second equation by 5 to get common coefficients of –15.
2. \( \begin{pmatrix} 4(9x - 10y = 7) \Rightarrow 36x - 40y = 28 \\ 5(5x + 8y = 31) \Rightarrow 25x + 40y = 155 \end{pmatrix} \)
3. (1, –3)
4. (10, –10)

Success for English Learners
1. –3
2. Multiply the first equation by 3 and the second by –2, or the first by –3 and the second by 2.
LESSON 8-5
Practice and Problem Solving: A/B

1. No solution; parallel lines with the same slopes.

2. Many solutions; same lines

3. \[ \begin{align*}
{y} &= -2x + 1 \\
{y} &= -2x - 3
\end{align*} \]
   - same slope but different \( y \)-intercepts, so no solution.

4. \[ \begin{align*}
{y} &= -5x + 2 \\
{y} &= 5x + 2
\end{align*} \]
   - different slopes, so one solution.

5. \[ \begin{align*}
{y} &= 3x - 2 \\
{y} &= 3x - 2
\end{align*} \]
   - same slope and \( y \)-intercepts, so many solutions.

6. The rates of deposit are the same, since the 2nd sister’s rate of $40 every 2 months is the same as the 1st sister’s rate of $20 every month. They start with the same amount, too. The total amounts of their savings will only vary for the months in which sister 1 puts her $20 in before sister 2 puts in $40 every 2 months.

Practice and Problem Solving: C

1. \( +3; -\frac{1}{3}; -\frac{1}{3} \)

2. \( y = 3x; x + 3y = 3; x + 3y = -9 \)

3. They are negative reciprocals.

4. They are the same.

5. \( \left( \frac{3}{10}, \frac{9}{10} \right) \)

6. Find the distances from the solution point to a point on line \( A \) and a point on line \( B \). Then, find the distance between the points on lines \( A \) and \( B \). Finally, check to see if the three distances satisfy the conditions for a right triangle, i.e. the square of the hypotenuse is equal to the sum of the squares of its two legs.

7. Yes, because the slopes, \( +3 \) and \( -\frac{1}{3} \), are negative reciprocals.

8. No, because the slopes are positive reciprocals.

9. Yes, because the slopes, \( \frac{3}{5} \) and \( -\frac{5}{3} \), are negative reciprocals.

Practice and Problem Solving: D

1. \( -2x; -2x; -2x; 6; -2x; 6; -1x; 3; \frac{5}{2}; \)
   - \( -\frac{1}{3}; \frac{5}{2} \)
   - \( 3x; 3; 3; -9; -9; -9; -1; 3; 9 \)
   - \( 2; -\frac{1}{3}; -\frac{1}{3} \)

3. \( 5; -\frac{5}{2}; -\frac{9}{3} \)

4. No solutions, because the lines have the same slope and different \( y \)-intercepts.

5. none

6. none

7. many
Reteach
1. many solutions
2. no solution
3. no solution

Reading Strategies
1. No solution. A false statement means that the lines are parallel.
2. One solution.

Success for English Learners
1. In one case, an untrue statement, $0 = 1$, results; in the other case, a true statement results, $0 = 0$.
2. An untrue outcome means that the system of equations has no solution. A true outcome means that the system has many solutions.

Module 8 Challenge
1. $c + e + p = 100$
2. $c + 2e + 3p = 200$
3. \[
\begin{align*}
c + e + p &= 100 \\
c + 2e + 3p &= 200
\end{align*}
\]
4. $e + 2p = 100$
5. Answers may vary. Sample answers: $(c, 10, 45), (c, 20, 40),$ and $(c, 30, 35)$
6. Answers may vary. Sample answers: $45, 20, 135; 40, 40, 120; 35, 60, 105$