Properties of Translations

Practice and Problem Solving: A/B

Describe the translation that maps point A to point A'.

1. _______________  

2. _______________

Draw the image of the figure after each translation.

3. 3 units left and 9 units down

4. 3 units right and 6 units up

5. a. Graph rectangle J'K'L'M', the image of rectangle JKL'M', after a translation of 1 unit right and 9 units up.

b. Find the area of each rectangle.

c. Is it possible for the area of a figure to change after it is translated? Explain.
Properties of Translations

The vertices of a figure are given. Draw the figure. Then draw its image after the described translation.

1. \(R(-4, 4), S(3, 4), T(3, 2)\)
   Translate 1 unit left and 6 units down.

2. \(A(-3, -7), B(7, -7), C(6, -3), D(0, -2)\)
   Translate 3 units left and 7 units up.

3. Figure \(ABCDEF\) is given.
   a. Translate \(ABCDEF\) 6 units left and 2 units down. What are the coordinates of \(A'B'C'D'E'F'\)?

   b. Translate \(A'B'C'D'E'F'\) 4 units down. What are the coordinates of \(A''B''C''D''E''F''\)?

   c. Translate \(A''B''C''D''E''F''\) 6 units right and 2 units up. What are the coordinates of \(A'''B'''C'''D'''E'''F'''\)?

   d. A pattern of a figure that repeats and covers a plane without overlapping and without gaps is called a tessellation. Can figure \(ABCDEF\) be translated to create a tessellation? Explain.

4. A translation of each point \((x, y)\) of a figure can be described using the coordinate notation \((x, y) \rightarrow (x + a, y + b)\), where \(a\) represents the horizontal distance moved and \(b\) represents the vertical distance moved. For triangle \(PQR\) with vertices \(P(-3, -1), Q(0, -1),\) and \(R(-1, -3)\), find the coordinates of the vertices of the image after the translation \((x, y) \rightarrow (x - 5, y + 7)\).
Properties of Translations

Answer the questions about the given translation of triangle $ABC$ onto triangle $A'B'C'$. The first one is done for you.

1. What is the image of $A(2, 4)$? ______________
2. What is the preimage of $B'(-3, 0)$? ______________
3. What is the image of $C(6, 1)$? ______________
4. Which side is congruent to side $AB$? ______________
5. Which angle is congruent to angle $C$? ______________
6. How would you describe the translation?

7. Quadrilateral $PQRS$ is given.
   a. Vertex $P$ is translated to point $P'$ as shown. Describe the translation.
      ______________________________________________________________________________________
   b. Use the same translation to translate the remaining vertices. Draw quadrilateral $P'Q'R'S'$.
   c. How do quadrilateral $PQRS$ and quadrilateral $P'Q'R'S'$ compare?
      ______________________________________________________________________________________

Draw the image of the figure after each translation.

8. 3 units right and 4 units down

9. 5 units left and 2 units up
Properties of Translations

Reteach

The description of a translation in a coordinate plane uses a combination of two translations – one translation slides the figure in a horizontal direction, and the other slides the figure in a vertical direction. An example is shown below.

Triangle $LMN$ is shown in the graph. The triangle can be translated 8 units right and 5 units down as shown below.

Step 1 Translate each vertex 8 units right.

Step 2 Translate each vertex 5 units down.

Step 3 Label the resulting vertices and connect them to form triangle $L'M'N'$.

Use a combination of two translations to draw the image of the figure.

1. Translate 6 units left and 7 units down.

2. Translate 7 units right and 9 units up.

3. When translating a figure using a combination of two translations, is the resulting figure congruent to the original figure? Explain.
Properties of Translations

A transformation is an operation that describes a change in the position, size, or shape of a figure. A translation is one kind of transformation.

In Exercises 1–5, use the translation shown.

1. What is the image of the translation?

2. What is the preimage of the translation?

3. How many vertices does the preimage have?

4. Is the image congruent to the preimage? Explain.

5. Describe the translation.

6. Describe the difference between a transformation and a translation.
Properties of Translations

Problem 1

Steps for Translating a Figure

Step 1 Apply the transformation rule to each vertex.

Step 2 Plot and label the new vertices on the same coordinate grid.

Step 3 Draw the resulting image by connecting the vertices.

Problem 2

1. Describe the translation in Problem 1.

2. In Problem 2, suppose triangle PQR is translated using the translation “4 units up and 3 units left.” Is the resulting image the same as in Problem 2? Explain why or why not.
Use the graph for Exercises 1–3.

1. Quadrilateral \( J \) is reflected across the \( x \)-axis. What is the image of the reflection?

2. Which two quadrilaterals are reflections of each other across the \( y \)-axis?

3. How are quadrilaterals \( H \) and \( J \) related?

Draw the image of the figure after each reflection.

4. across the \( x \)-axis

5. across the \( y \)-axis

6. a. Graph rectangle \( K'L'M'N' \), the image of rectangle \( KLMN \) after a reflection across the \( y \)-axis.

   b. What is the perimeter of each rectangle?

   c. Is it possible for the perimeter of a figure to change after it is reflected? Explain.
The vertices of a figure are given. Draw the figure. Then draw its image after the described reflection.

1. \(W(-5, 2), X(3, 0), Y(-2, -5)\)
   Reflect across the \(x\)-axis.

2. \(G(3, -3), H(-5, -1), J(-4, 3), K(2, 2)\)
   Reflect across the \(y\)-axis.

3. Triangle \(ABC\) is reflected across the \(y\)-axis to form triangle \(A'B'C'\).
   The coordinates of the vertices of the triangles are given below.

   **Triangle \(ABC\):** 
   \(A(2, 3)\) \(B(6, 7)\) \(C(4, 1)\)

   **Triangle \(A'B'C'\):** 
   \(A'(-2, 3)\) \(B'(-6, 7)\) \(C'(-4, 1)\)

   Make a conjecture about the coordinates of a figure and its image after a reflection across the \(y\)-axis.

   ________________________________________________________________________________
   ________________________________________________________________________________

Draw the image of the given figure after the two transformations.

4. Translate 8 units right and 1 unit up. Reflect across the \(x\)-axis.

5. Reflect across the \(y\)-axis. Translate 2 units left and 5 units up.
Properties of Reflections

Practice and Problem Solving: D

Answer the questions about the given reflection of quadrilateral \(ABCD\) onto quadrilateral \(A'B'C'D'\). The first one is done for you.

1. What is the image of \(A(-6, 2)\)? \(A'(6, 2)\)

2. What is the preimage of \(B'(5, 6)\)?

3. What is the image of \(C(-3, 7)\)?

4. Which side is congruent to side \(CD\)?

5. Which angle is congruent to angle \(D\)?

6. How would you describe the reflection?

Draw the image of the figure after each reflection.

7. across the \(x\)-axis

8. across the \(y\)-axis

Choose the word in parentheses that makes the statement true. The first one is done for you.

9. A reflection is a transformation that (slides, flips, turns) a figure across a line. flips

10. The image of a reflection is (sometimes, always, never) congruent to the original figure.

11. In a reflection across the \(x\)-axis, the \((x\)-coordinate, \(y\)-coordinate) changes.
Properties of Reflections

Reteach

You can use tracing paper to reflect a figure in the coordinate plane. The graphs below show how to reflect a triangle across the y-axis.

Start by tracing the figure and the axes on tracing paper.

Flip the tracing paper over, making sure to align the axes. Transfer the flipped image onto the coordinate plane.

As shown above, flip the paper horizontally for a reflection in the y-axis. For a reflection in the x-axis, flip the paper vertically.

Use tracing paper to draw the image after the reflection.

1. across the y-axis
2. across the x-axis
Properties of Reflections

Reading Strategies: Analyze Graphs

A transformation is an operation that describes a change in the position, size, or shape of a figure. A reflection is one kind of transformation.

In Exercises 1–4, use the reflection shown.

1. What is the image of the reflection?

2. What is the original figure of the reflection?

3. Name a pair of corresponding vertices.

4. Describe the reflection.

5. Quadrilateral \( P'Q'R'S' \) is the image of a figure that is flipped across the \( x \)-axis. Complete each statement about the transformation.

   a. The name of the original figure is __________________________.

   b. The transformation is called a __________________________.

6. Suppose a figure is reflected across a line. Describe the relationship between a point on the original figure and its corresponding point on the image.
Problem 1

Problem 2

Steps for Reflecting the Vertices of a Figure Across a Line

1. Describe the reflection in Problem 1.

2. In Problem 2, what other step is needed to complete the reflection of the original figure?
Properties of Rotations

Practice and Problem Solving: A/B

Use the figures at the right for Exercises 1–5. Triangle A has been rotated about the origin.

1. Which triangle shows a 90° counterclockwise rotation? __________
2. Which triangle shows a 180° counterclockwise rotation? __________
3. Which triangle shows a 270° clockwise rotation? __________
4. Which triangle shows a 270° counterclockwise rotation? __________
5. If the sides of triangle A have lengths of 30 cm, 40 cm, and 50 cm, what are the lengths of the sides of triangle D?

Use the figures at the right for Exercises 6–10. Figure A is to be rotated about the origin.

6. If you rotate figure A 90° counterclockwise, what quadrant will the image be in? __________
7. If you rotate figure A 270° counterclockwise, what quadrant will the image be in? __________
8. If you rotate figure A 180° clockwise, what quadrant will the image be in? __________
9. If you rotate figure A 360° clockwise, what quadrant will the image be in? __________
10. If the measures of two angles in figure A are 60° and 120°, what will the measure of those two angles be in the rotated figure?

Use the grid at the right for Exercises 11–12.

11. Draw a square to show a rotation of 90° clockwise about the origin of the given square in quadrant I.
12. What other transformation would result in the same image as you drew in Exercise 11?
Properties of Rotations

Tell whether each triangle shows a rotation of triangle $A$ about the origin. If a rotation is shown, give the number of degrees and the direction of the rotation. If a rotation is not shown, explain why it is not a rotation.

1. Triangle $B$

2. Triangle $C$

3. Triangle $D$

Solve.

4. You rotate an equilateral triangle clockwise $60^\circ$ as shown at the right. If you continue to make $60^\circ$ rotations, what larger figure will be formed?

5. You draw square $S$ with one vertex at the origin and one side along the $x$-axis. You rotate square $S$ clockwise about the origin. You rotate square $S$ through $90^\circ$, $180^\circ$, and $270^\circ$. Describe the large figure that is formed by the four squares. Include the size of the sides in relation to square $S$.

Draw the image of the figure after the given rotation about the origin.

6. $180^\circ$

7. $90^\circ$ counterclockwise
LESSON 9-3 Properties of Rotations

Practice and Problem Solving: D

Use the figures at the right for Exercises 1–5. Rectangle A has been rotated about the origin. The first one has been done for you.

1. Which rectangle shows a 270° counterclockwise rotation? B
2. Which rectangle shows a 180° clockwise rotation?
3. Which rectangle shows a 90° clockwise rotation?
4. Which rectangle shows a 90° counterclockwise rotation?
5. If two sides of rectangle A have lengths of 2 cm and 4 cm, what are the lengths of the two corresponding sides of rectangle C?

Use the figure at the right for Exercises 6–9. Figure A is to be rotated about the origin. The first one has been done for you.

6. If you rotate figure A 270° clockwise, what quadrant will the image be in?
7. If you rotate figure A 90° counterclockwise, what quadrant will the image be in?
8. If you rotate figure A 90° clockwise, what quadrant will the image be in?
9. If you rotate figure A 180° clockwise, what quadrant will the image be in?

Use the grid at the right for Exercises 10–11.

10. Draw a triangle to show a rotation of 180° clockwise about the origin of triangle J. Label the image triangle as K.

11. Suppose you rotate triangle J 180° counterclockwise about the origin. Compare the image of that rotation with triangle K.
Properties of Rotations

**Reteach**

A rotation is a change in position of a figure.

A rotation will turn the figure around a point called the center of rotation.

A rotation does not change the size of the figure.

At the right, triangle ABC has been rotated 90° clockwise.

The resulting figure is triangle A'B'C'.

Below are two more rotations of triangle ABC.

90° counterclockwise rotation

180° clockwise rotation

Use the figures at the right to answer each question.

**Triangle A has been rotated about the origin.**

1. Which triangle shows a 90° counterclockwise rotation?

2. Which triangle shows a 180° clockwise rotation?

3. Which triangle shows a 90° clockwise rotation?

4. Which triangle shows a 180° counterclockwise rotation?

5. If the sides of triangle A have lengths of 3 cm, 4 cm, and 5 cm, what are the lengths of the sides of triangle B?

6. Explain why the answers to Exercises 2 and 4 are the same.
Properties of Rotations

Reading Strategies: Using Visual Clues

Rotate the arrow 90° counterclockwise about the origin.

In which quadrant will the image be located?
Think: Counterclockwise goes from 12 to 9. The image will be in quadrant II.

How will the shape turn? What will it look like?
Think: Go from 12 to 9 on a clock.

Where will the image be on the grid?
Think: Any vertices at the origin? No
Any vertices on an axis? If so, which axis will the image be on and where? One side is from 1 to 2 on the x-axis. The image will be from 1 to 2 on the y-axis.

What else can help me?
Think: The point of the arrow is at (1.5, 4). The image will be in quadrant II, 1.5 units up from the x-axis and 4 units from the y-axis.

Use your clues to draw the image.

Rotate the figure on the grid 90° clockwise.

1. Write the clues you can use.

2. Draw the image of the figure after a 90° clockwise rotation.
A rotation turns a figure around a point called the center of rotation.

**Problem 1**

Rotate the triangle 90° clockwise around the origin.

Clockwise moves like clock hands.

Start with the triangle.

Each vertex turns 90°.

**Problem 2**

Rotate the triangle 180° counterclockwise around the origin.

Counterclockwise moves opposite of clock hands.

Start with the triangle.

Each vertex turns 180°.

Describe each rotation by giving the angle measure and the direction of rotation. The rotated image is shown in gray.

1. ____________________________ 2. ____________________________
Write an algebraic rule to describe each transformation of figure A to figure A′. Then describe the transformation.

1. ___________________________________________________________________________________

2. ___________________________________________________________________________________

Use the given rule to graph the image of each figure. Then describe the transformation.

3. \((x, y) \rightarrow (-x, y)\)  

4. \((x, y) \rightarrow (-x, -y)\)  

Solve.

5. Triangle \(ABC\) has vertices \(A(2, -1)\), \(B(-3, 0)\), and \(C(-1, 4)\). Find the vertices of the image of triangle \(ABC\) after a translation of 2 units up.

_________________________________________________________________________________________

6. Triangle \(LMN\) has \(L\) at \((1, -1)\) and \(M\) at \((2, 3)\). Triangle \(L'M'N'\) has \(L'\) at \((-1, -1)\) and \(M'\) is at \((3, -2)\). Describe the transformation.

_________________________________________________________________________________________
Write an algebraic rule to describe each transformation of figure A to figure $A'$. Then describe the transformation.

1. ____________________________________________  ____________________________________________

2.  ____________________________________________  ____________________________________________

Use the given rule to graph the image of each figure. Then describe the transformation.

3. $(x, y) \rightarrow (x, -y)$  4. $(x, y) \rightarrow (-x, -y)$

Solve.

5. Triangle $ABC$ has vertices $A(2, -1)$, $B(0, 0)$, and $C(-1, 4)$. State a rule for an algebraic transformation where vertex $B$ will not be at the origin.

6. Triangle $LMN$ has $L$ at $(1, -1)$ and $M$ at $(2, 3)$. Triangle $L'M'N'$ has $L'$ at $(-1, -1)$, $M'$ is at $(3, -2)$, and $N'$ is at $(-3, 0)$. What are the coordinates of vertex $N'$? Describe the transformation.
LESSON 9-4

Algebraic Representations of Transformations

Practice and Problem Solving: D

Complete the algebraic rule to describe each transformation of figure A to figure $A'$. Then complete the description of the transformation. The first one is done for you.

1. $(x, y) \rightarrow (\text{___}, \text{___})$
   
   rotation of $\text{___}^{\circ}$ clockwise

2. $(x, y) \rightarrow (\text{___})$

   translation $\text{___}$ units

3. $(x, y) \rightarrow (\text{___})$

   reflection over the ___-axis

4. $(x, y) \rightarrow (\text{___})$

   rotation of $\text{___}^{\circ}$

Figure A is shown on the right. Use the rule $(x, y) \rightarrow (x, -y)$ to give the coordinates of the image of figure A. The first one is done for you.

5. $(-4, -4) \rightarrow (-4, \text{___})$

6. $(-1, -4) \rightarrow (\text{___}, \text{___})$

7. $(-2, -1) \rightarrow (\text{___}, \text{___})$

8. Graph the image of figure A for the rule $(x, y) \rightarrow (x, -y)$.
A **transformation** is a change in size or position of a figure. The transformations below change only the position of the figure, not the size.

- A **translation** will *slide* the figure horizontally and/or vertically.
- A **reflection** will *flip* the figure across an axis.
- A **rotation** will *turn* the figure around the origin.

This table shows how the coordinates change with each transformation.

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Coordinate Mapping</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Translation</strong></td>
<td></td>
</tr>
<tr>
<td>((x, y) \rightarrow (x + a, y + b)) translates left or right (a) units and up or down (b) units</td>
<td></td>
</tr>
<tr>
<td><strong>Reflection</strong></td>
<td></td>
</tr>
<tr>
<td>((x, y) \rightarrow (−x, y)) reflects across the (y)-axis</td>
<td></td>
</tr>
<tr>
<td>((x, y) \rightarrow (x, −y)) reflects across the (x)-axis</td>
<td></td>
</tr>
<tr>
<td><strong>Rotation</strong></td>
<td></td>
</tr>
<tr>
<td>((x, y) \rightarrow (−x, −y)) rotates 180° around origin</td>
<td></td>
</tr>
<tr>
<td>((x, y) \rightarrow (y, −x)) rotates 90° clockwise around origin</td>
<td></td>
</tr>
<tr>
<td>((x, y) \rightarrow (−y, x)) rotates 90° counterclockwise around origin</td>
<td></td>
</tr>
</tbody>
</table>

A triangle with coordinates of \((0, 0), (1, 4),\) and \((3, −2)\) is transformed so the coordinates are \((0, 0), (−4, 1),\) and \((2, 3)\). What transformation was performed?

Analyze each corresponding pairs of coordinates:

- \((0, 0)\) to \((0, 0)\)  
  Think: Could be reflection or rotation since \(0 = −0\).
- \((1, 4)\) to \((−4, 1)\)  
  Think: Since \(x\) and \(y\) are interchanged, it is a rotation and \(y\) changes sign, so it is a 90° counterclockwise rotation around origin.

**Identify the transformation from the original figure to the image.**

1. Original: \(A(−2, −4), B(5, 1), C(5, −4)\)  
   Image: \(A′(2, −4), B′(−5, 1), C′(−5, −4)\)

2. Original: \(A(−8, 2), B(−4, 7), C(−7, 2)\)  
   Image: \(A′(−2, −8), B′(−7, −4), C′(−2, −7)\)

3. Original: \(A(3, 4), B(−1, 2), C(−3, −5)\)  
   Image: \(A′(3, 8), B′(−1, 6), C′(−3, −1)\)

4. Original: \(A(1, 1), B(2, −2), C(4, 3)\)  
   Image: \(A′(−1, −1), B′(−2, 2), C′(−4, −3)\)

5. Original: \(A(−5, −6), B(−2, 4), C(3, 0)\)  
   Image: \(A′(−5, 6), B′(−2, −4), C′(3, 0)\)

Original content Copyright © by Houghton Mifflin Harcourt. Additions and changes to the original content are the responsibility of the instructor.
A transformation is a change in size or position of a figure. The transformations below change only the position of the figure, not the size.

- A translation will slide the figure horizontally and/or vertically.
- A reflection will flip the figure across an axis.
- A rotation will turn the figure around the origin.

\[(x, y) \rightarrow (x + a, y + b)\]
Analysis: Each value of \(x\) and/or \(y\) changes by a certain amount, \(a\) for \(x\) and \(b\) for \(y\).
Transformation: Translation over \(a\) units and/or up or down \(b\) units

\[(x, y) \rightarrow (-x, -y)\]
Analysis: The sign of both \(x\) and \(y\) change.
Transformation: Rotation of 180° around origin

\[(x, y) \rightarrow (-x, y)\] and \[(x, y) \rightarrow (x, -y)\]
Analysis: The sign of either \(x\) or \(y\) changes.
Transformation: Reflection across the \(y\)-axis if the sign of \(x\) changes
Reflection across the \(x\)-axis if the sign of \(y\) changes

\[(x, y) \rightarrow (y, -x)\] and \[(x, y) \rightarrow (-y, x)\]
Analysis: The coordinates are switched and the sign of one changes.
Transformation: Rotation of 90° clockwise if the sign of \(x\) changes
Rotation of 90° counterclockwise if the sign of \(y\) changes

Identify the transformation from the original figure to the image.

1. Original: \( A(-1, -4), B(5, 1), C(5, -4) \)
   Image: \( A'(1, -4), B'(7, 1), C'(7, -4) \)

2. Original: \( A(6, 2), B(-4, 2), C(-1, 4) \)
   Image: \( A'(2, -6), B'(2, 4), C'(-4, 1) \)

3. Original: \( A(3, -4), B(-1, 2), C(3, -5) \)
   Image: \( A'(-3, 4), B'(1, -2), C'(-3, 5) \)

4. Original: \( A(1, 1), B(2, -2), C(4, 3) \)
   Image: \( A'(-1, 1), B'(-2, -2), C'(-4, 3) \)

Original content Copyright © by Houghton Mifflin Harcourt. Additions and changes to the original content are the responsibility of the instructor.
**Algebraic Representations of Transformations**

**Success for English Learners**

Some transformations change the position of a figure, but do not change the size of the figure.

**Problem 1**

How the positions change

<table>
<thead>
<tr>
<th>Translation</th>
<th>Reflection</th>
<th>Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Think: <strong>Slide</strong></td>
<td>Think: <strong>Flip</strong></td>
<td>Think: <strong>Turn</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Translation</th>
<th>Reflection</th>
<th>Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x, y) \rightarrow (x + a, y + b)$ translates left or right $a$ units and up or down $b$ units</td>
<td>$(x, y) \rightarrow (-x, y)$ reflects across the $y$-axis</td>
<td>$(x, y) \rightarrow (-x, -y)$ rotates $180^\circ$ around origin</td>
</tr>
<tr>
<td>$(x, y) \rightarrow (x, -y)$ reflects across the $x$-axis</td>
<td>$(x, y) \rightarrow (y, -x)$ rotates $90^\circ$ clockwise around origin</td>
<td>$(x, y) \rightarrow (-y, x)$ rotates $90^\circ$ counterclockwise around origin</td>
</tr>
</tbody>
</table>

**Problem 2**

How the coordinates change

<table>
<thead>
<tr>
<th>Translation</th>
<th>Reflection</th>
<th>Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Think: <strong>Slide</strong></td>
<td>Think: <strong>Flip</strong></td>
<td>Think: <strong>Turn</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Translation</th>
<th>Reflection</th>
<th>Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x, y) \rightarrow (x + a, y + b)$ translates left or right $a$ units and up or down $b$ units</td>
<td>$(x, y) \rightarrow (-x, y)$ reflects across the $y$-axis</td>
<td>$(x, y) \rightarrow (-x, -y)$ rotates $180^\circ$ around origin</td>
</tr>
<tr>
<td>$(x, y) \rightarrow (x, -y)$ reflects across the $x$-axis</td>
<td>$(x, y) \rightarrow (y, -x)$ rotates $90^\circ$ clockwise around origin</td>
<td>$(x, y) \rightarrow (-y, x)$ rotates $90^\circ$ counterclockwise around origin</td>
</tr>
</tbody>
</table>

Tell which transformation was used to change the position of the first arrow so it looks like the second arrow.

1. $\Rightarrow \rightarrow \Leftarrow$

2. $\Leftarrow \rightarrow \Leftarrow$

3. $\Uparrow \rightarrow \Downarrow$

Tell which transformation is shown by the change in ordered pairs.

4. $(3, 4), (2, -1), (0, 0) \rightarrow (4, -3), (-1, -2), (0, 0)$

5. $(-2, 0), (0, -2), (2, 2) \rightarrow (0, 0), (2, -2), (4, 2)$

6. $(1, 5), (1, -1), (-3, -2) \rightarrow (1, -5), (1, 1), (-3, 2)$
**LESSON 9-5**

**Congruent Figures**

**Practice and Problem Solving: A/B**

Identify a sequence of transformations that will transform figure A into figure C.

1. What transformation is used to transform figure A to figure B?
   __________________________________________

2. What transformation is used to transform figure B to figure C?
   __________________________________________

3. What sequence of transformations is used to transform figure A to figure C? Express the transformations algebraically.
   _______________________________________________________________________________________

Complete each transformation.

4. Transform figure A by reflecting it over the y-axis. Label the new figure, B.

5. Transform figure B to figure C by applying (x, y) → (x, y + 5).

6. Transform figure C to figure D by rotating it 90° counterclockwise around the origin.

7. Compare figure A with figure D. Are the two figures congruent? ________________

8. Do figures A and D have the same or different orientation? ________________

Alice wanted a pool in location A on the map at the right. However, underground wires forced her to move the pool to location B.

9. What transformations were applied to the pool at location A to move it to location B?
   _______________________________________________________________________________________
   _______________________________________________________________________________________

10. Did the relocation change the size or orientation of the pool?

________________________________________________________________________________________
**LESSON 9-5**

**Congruent Figures**

*Practice and Problem Solving: C*

A transformation of figure \(A\) produces figure \(B\). A second transformation produces figure \(C\).

1. What sequence of transformations is used to transform figure \(A\) to figure \(C\)? Express the transformations algebraically.

______________________________________________________________________________

______________________________________________________________________________

Triangle \(A\) was transformed to triangle \(B\).
Triangle \(B\) was transformed to triangle \(C\).
Triangle \(C\) was transformed to triangle \(D\).

2. Each transformation included the same two transformations. Express those two transformations algebraically.

______________________________________________________________________________

______________________________________________________________________________

3. Will the same two transformations applied to triangle \(D\) yield triangle \(A\)? _______________

4. What shape is formed by the hypotenuses of the right triangles? Explain how you know.

_________________________________________________________________________________________

Alice wanted a pool in location \(A\) on the map at the right. However, underground wires forced her to move the pool to location \(B\).

5. What transformations were applied to the pool at location \(A\) to move it to location \(B\)?

______________________________________________________________________________

______________________________________________________________________________

6. Alice still was not happy. She wants the pool to have the same orientation as pool \(A\). Describe possible transformations that would locate the pool so that it does not touch the wires and has the same orientation as pool \(A\). Draw the location that shows your suggested transformations.

_________________________________________________________________________________________
**LESSON 9-5**

**Congruent Figures**

**Practice and Problem Solving: D**

Identify a sequence of transformations that will transform figure A into figure C. The first one is done for you.

1. What transformation is used to transform figure A to figure B?
   
   **rotation 90° counterclockwise**

2. What transformation is used to transform figure B to figure C?
   
   __________________________________________

3. What sequence of transformations is used to transform figure A to figure C? Express the transformations algebraically.
   
   __________________________________________

   __________________________________________

Complete each transformation. The first one is done for you.

4. Transform figure A to figure B by rotating it 90° clockwise around the origin, so \((x, y) \rightarrow (y - x)\).

5. Compare figure A with figure B.
   Are the two figures congruent? ____________

6. Do figures A and B have the same or different orientation? ____________

7. Transform figure B to figure C by translating it 8 units right, so \((x, y) \rightarrow (x + 8, y)\).

8. Compare figure B with figure C.
   Are the two figures congruent? ____________

9. Do figures B and C have the same or different orientation? ____________

10. Transform figure C to figure D by reflecting it over the x-axis so \((x, y) \rightarrow (x, -y)\).

11. Compare figure C with figure D. Are the two figures congruent? ____________

12. Do figures C and D have the same or different orientation? ____________

13. Compare figure A with figure D. Are the two figures congruent? ____________

14. Do figures A and D have the same or different orientation? ____________

Original content Copyright © by Houghton Mifflin Harcourt. Additions and changes to the original content are the responsibility of the instructor.
When combining the transformations below, the original figure and transformed figure are **congruent**. Even though the size does not change, the orientation of the figure might change.

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Algebraic Coordinate Mapping</th>
<th>Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translation</td>
<td>((x, y) \rightarrow (x + a, y + b)) translates left or right (a) units and up or down (b) units</td>
<td>same</td>
</tr>
</tbody>
</table>
| Reflection     | \((x, y) \rightarrow (-x, y)\) reflects across the y-axis  
\((x, y) \rightarrow (x, -y)\) reflects across the x-axis | different |
| Rotation       | \((x, y) \rightarrow (-x, -y)\) rotates 180° around origin  
\((x, y) \rightarrow (y, -x)\) rotates 90° clockwise around origin  
\((x, y) \rightarrow (-y, x)\) rotates 90° counterclockwise around origin | different |

1st transformation: translation right 4 units  
\((x, y) \rightarrow (x + 4, y)\),  
orientation: same

2nd transformation: reflection over the x-axis  
\((x, y) \rightarrow (x, -y)\),  
orientation: different

3rd transformation: rotation 90° clockwise  
\((x, y) \rightarrow (y, -x)\)  
orientation: different

Describe each transformation. Express each algebraically.  
Tell whether the orientation is the same or different.

1. First transformation
   - Description: __________________________________________
   - Algebraically: _________________________________________
   - Orientation: _________________________________________

2. Second transformation
   - Description: __________________________________________
   - Algebraically: _________________________________________
   - Orientation: _________________________________________
Some transformations create congruent figures. These transformations are translations, rotations, and reflections.

Identify the transformation:
Translation 5 units right
Compare and contrast:
Each value of $x$ changes by 5.
Algebraic notation:
$$(x, y) \rightarrow (x + 5, y)$$
The figures are congruent.

Identify the transformation:
Reflection over the $y$-axis
Compare and contrast:
The value of $x$ changes signs.
Algebraic notation:
$$(x, y) \rightarrow (-x, y)$$
The figures are congruent.

Identify the transformation:
90° rotation clockwise
Compare and contrast:
The $x$ and $y$ are switched, and $x$ changes signs.
Algebraic notation:
$$(x, y) \rightarrow (y, -x)$$
The figures are congruent.

Use the figure for Exercises 1–4.
1. Identify the transformation. __________________________
2. Compare and contrast changes. __________________________
3. Algebraic notation: $(x, y) \rightarrow$ __________________________
4. Are the figures congruent? __________________________

Use the figure for Exercises 5–8.
5. Identify the transformation. __________________________
6. Compare and contrast changes. __________________________
7. Algebraic notation: $(x, y) \rightarrow$ __________________________
8. Are the figures congruent? __________________________
Some transformations change the position of a shape but do not change its size. Translations, reflections, and rotations all produce congruent figures.

**Problem 1**

**How the positions change**

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Think:</th>
<th>Translation</th>
<th>Reflection</th>
<th>Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Slide</td>
<td>Flip over a line</td>
<td>Turn about a point</td>
</tr>
<tr>
<td></td>
<td></td>
<td><img src="slide.png" alt="Translation Diagram" /></td>
<td><img src="flip.png" alt="Reflection Diagram" /></td>
<td><img src="turn.png" alt="Rotation Diagram" /></td>
</tr>
</tbody>
</table>

**Problem 2**

**How the coordinates change**

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Think:</th>
<th>Translation</th>
<th>Reflection</th>
<th>Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Slide</td>
<td>Flip</td>
<td>Turn</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(x, y) \rightarrow (x + a, y + b)$ translates left or right $a$ units and up or down $b$ units</td>
<td>$(x, y) \rightarrow (-x, y)$ reflects across the $y$-axis</td>
<td>$(x, y) \rightarrow (-x, -y)$ rotates $180^\circ$ around origin</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(x, y) \rightarrow (x, -y)$ reflects across the $x$-axis</td>
<td>$(x, y) \rightarrow (y, -x)$ rotates $90^\circ$ clockwise around origin</td>
<td>$(x, y) \rightarrow (-y, x)$ rotates $90^\circ$ counterclockwise around origin</td>
</tr>
</tbody>
</table>

Write the name of each kind of transformation. Use $x$ and $y$ to describe how the coordinates change.

1.  
2.  
3.  

Original content Copyright © by Houghton Mifflin Harcourt. Additions and changes to the original content are the responsibility of the instructor.
Transformations and Congruence

Challenge

In this module the transformations you dealt with are called isometric transformations. Isometric means "has the same measures" so these transformations produce congruent images. There are two types of isometric transformations:

Direct—the original figure and the image have the same size, shape, and orientation. If the figure is named by letters they are not reversed.

Opposite—the original figure and the image have the same size and shape, but the orientation is not preserved. The image may point in a different direction and lettered points may be reversed.

For each of Exercises 1–4 write an algebraic rule for the coordinates of the image. Then identify the type of transformation and state whether it is direct or opposite.

1. ____________________________________________________________________________

2. ____________________________________________________________________________

3. ____________________________________________________________________________

4. ____________________________________________________________________________

Look at the coordinate pairs for each type of isometric transformation.

5. Which transformations are direct?

_______________________________________________________________________________

6. Which transformations are opposite?

_______________________________________________________________________________

Original content Copyright © by Houghton Mifflin Harcourt. Additions and changes to the original content are the responsibility of the instructor.
Use triangles \( ABC \) and \( A'B'C' \) for Exercises 1–4.

1. Use the coordinates to find the lengths of the sides.

   Triangle \( ABC \): \( AB = \) ___; \( BC = \) ___

   Triangle \( A'B'C' \): \( A'B' = \) ___; \( B'C' = \) ___

2. Find the ratios of the corresponding sides.

   \[
   \frac{A'B'}{AB} = \quad \frac{B'C'}{BC} = \quad \frac{A'B'}{AB} = \quad \frac{B'C'}{BC} = \quad
   \]

3. Is triangle \( A'B'C' \) a dilation of triangle \( ABC \)? ____________

4. If triangle \( A'B'C' \) is a dilation of triangle \( ABC \), is it a reduction or an enlargement? ____________

For Exercises 5–8, tell whether one figure is a dilation of the other or not. If one figure is a dilation of the other, tell whether it is an enlargement or a reduction. Explain your reasoning.

5. Triangle \( R'S'T' \) has sides of 3 cm, 4 cm, and 5 cm. Triangle \( RST \) has sides of 12 cm, 16 cm, and 25 cm.

   __________________________________________________________________________________________
   __________________________________________________________________________________________

6. Quadrilateral \( WBCD \) has coordinates of \( W(0, 0), B(0, 4), C(-6, 4), \) and \( D(-6, 0) \). Quadrilateral \( W'B'C'D' \) has coordinates of \( W(0, 0), B'(0, 2), C'(-3, 2), \) and \( D'(-3, 0) \).

   __________________________________________________________________________________________
   __________________________________________________________________________________________

7. Triangle \( MLQ \) has sides of 4 cm, 4 cm, and 7 cm. Triangle \( M'L'Q' \) has sides of 12 cm, 12 cm, and 21 cm.

   __________________________________________________________________________________________
   __________________________________________________________________________________________

8. Do the figures at the right show a dilation? Explain.

   __________________________________________________________________________________________
   __________________________________________________________________________________________
   __________________________________________________________________________________________
Properties of Dilations

Practice and Problem Solving: C

Identify the scale factor used in each dilation.

1. ![Diagram 1]
   scale factor: ___

2. ![Diagram 2]
   scale factor: ___

Find the center of dilation for each pair of figures.

3. ![Diagram 3]

4. ![Diagram 4]

Solve.

5. A rectangle on the coordinate plane has vertices at (0, 0), (3, 0), (3, 2),
   and (0, 2). A dilation of the rectangle has vertices at (0, 0), (9, 0),
   (9, 6), and (0, 6). Find the scale factor and area of each rectangle.

   scale factor: ____ ; area of original rectangle: ____ ; area of dilation: ____

6. A rectangle on the coordinate plane has vertices at (0, 0), (4, 0), (4, 2),
   and (0, 2). A dilation of the rectangle has vertices at (0, 0), (2, 0),
   (2, 1), and (0, 1). Find the scale factor and area of each rectangle.

   scale factor: ____ ; area of original rectangle: ____ ; area of dilation: ____

7. Use the answers to Exercises 5 and 6. Make a conjecture about the
   relationship of the scale factor to the area of an original rectangle and
   its dilation.

   ________________________________________________
   ________________________________________________
Properties of Dilations

LESSON 10-1

Practice and Problem Solving: D

Use triangles $ABC$ and $A'B'C'$ for Exercises 1–4. The first one is done for you.

1. Use the coordinates to find the lengths of the sides.
   - Triangle $ABC$: $AC = \underline{3}$; $BC = \underline{2}$
   - Triangle $A'B'C'$: $A'C' = \underline{9}$; $B'C' = \underline{6}$

2. Find the ratios of the corresponding sides.
   - $\frac{A'C'}{AC} = \underline{\text{____}}$; $\frac{B'C'}{BC} = \underline{\text{____}}$

3. Is triangle $A'B'C'$ a dilation of triangle $ABC$?    YES    NO

4. If triangle $A'B'C'$ is a dilation of triangle $ABC$, is it a reduction or an enlargement? ________________

Use rectangles $ABCD$ and $A'B'C'D'$ for Exercises 5–8.

5. Use the coordinates to find the lengths of the sides.
   - Rectangle $ABCD$: $AB = \underline{\text{____}}$; $BC = \underline{\text{____}}$; $CD = \underline{\text{____}}$; $DA = \underline{\text{____}}$
   - Rectangle $A'B'C'D'$: $A'B' = \underline{\text{____}}$; $B'C' = \underline{\text{____}}$; $C'D' = \underline{\text{____}}$; $D'A' = \underline{\text{____}}$

6. Find the ratios of the corresponding sides.
   - $\frac{A'B'}{AB} = \underline{\text{____}}$; $\frac{B'C'}{BC} = \underline{\text{____}}$
   - $\frac{C'D'}{CD} = \underline{\text{____}}$; $\frac{D'A'}{DA} = \underline{\text{____}}$

7. Is rectangle $A'B'C'D'$ a dilation of rectangle $ABCD$?    YES    NO

8. If rectangle $A'B'C'D'$ is a dilation of rectangle $ABCD$, is it a reduction or an enlargement? ________________

Solve.

9. If the scale factor is greater than one, is the new figure an enlargement or a reduction of the original figure? ________________
Properties of Dilations

Reteach

A dilation can change the size of a figure without changing its shape.

Lines drawn through the corresponding vertices meet at a point called the center of dilation.

To determine whether a transformation is a dilation, compare the ratios of the lengths of the corresponding sides.

\[
\frac{A'B'}{AB} = \frac{2}{1} = 2 \\
\frac{B'C'}{BC} = \frac{6}{3} = 2
\]

The ratios are equal, so the triangles are similar, and the transformation is a dilation.

Determine whether each transformation is a dilation.

1. 

\[
\frac{E'F'}{EF} = __ = __ \\
\frac{F'G'}{FG} = __ = __
\]

Are the ratios equal? ________________

Is this a dilation? ________________

2. 

\[
\frac{P'R'}{PR} = __ = __ \\
\frac{P'S'}{PS} = __ = __
\]

Are the ratios equal? ________________

Is this a dilation? ________________
Properties of Dilations

Reading Strategies: Use a Visual Aid

A dilation changes the size, but not the shape, of a figure. It is a special type of transformation. A figure can be enlarged or reduced through dilation.

The shaded triangle is the original figure. This dilation is an enlargement of the triangle. The triangles have the same shape. Only the size has changed.

If you draw lines through the corresponding vertices of the original figure and its dilation, the lines all intersect at a point. In this dilation, that point is the origin.

The shaded rectangle is the original figure. This dilation is a reduction of the original rectangle.

Notice the dashed lines connect the corresponding vertices and intersect at a single point. The point of intersection does not always have to be at the origin.

Tell whether each transformation is a dilation. Write yes or no.

1. [Image of a dilation example]

2. [Image of a dilation example]
Properties of Dilations
Success for English Learners

A dilation changes the size of a figure without changing its shape. Some dilations are enlargements. Some dilations are reductions. In a dilation, the ratios of the lengths of corresponding sides are equal.

**Problem 1**

Do the figures at the right show a dilation? If so, is it an enlargement or a reduction?

\[
\begin{align*}
\frac{A'C'}{AC} &= \frac{5}{2} = 2.5 \\
\frac{B'C'}{BC} &= \frac{5}{2} = 2.5
\end{align*}
\]

These are equal. This is a dilation. 

\[A'B'C'\] is an enlargement of \(ABC\) because:

- the ratio of corresponding lengths is greater than 1, and
- \(A'B'C'\) is the same shape as \(ABC\), but larger.

**Problem 2**

Do the figures at the right show a dilation? If so, is it an enlargement or a reduction?

\[
\begin{align*}
\frac{A'D'}{AD} &= \frac{2}{6} = \frac{1}{3} \\
\frac{B'C'}{BC} &= \frac{1}{3}
\end{align*}
\]

These are equal. This is a dilation. 

\[A'B'C'D'\] is a reduction of \(ABCD\) because:

- the ratio of corresponding lengths is less than 1, and
- \(A'B'C'D'\) is the same shape as \(ABCD\), but smaller.

Tell whether each dilation is an enlargement or a reduction.

1. 

2. 

Original content Copyright © by Houghton Mifflin Harcourt. Additions and changes to the original content are the responsibility of the instructor.
Algebraic Representations of Dilations

Practice and Problem Solving: A/B

Use triangle $ABC$ for Exercises 1–4.

1. Give the coordinates of each vertex of $\triangle ABC$.
   
   $A$_________________ $B$_________________ $C$_________________

2. Multiply each coordinate of the vertices of $\triangle ABC$ by 2 to find the vertices of the dilated image $\triangle A'B'C'$.
   
   $A'$_________________ $B'$_________________ $C'$_________________

3. Graph $\triangle A'B'C'$.

4. Complete this algebraic rule to describe the dilation.
   
   $(x, y) \rightarrow$___________________________

Use the figures at the right for Exercises 5–7.

5. Give the coordinates of each vertex of figure $JKLM$.
   
   $J$_________________ $K$_________________ $L$_________________ $M$_________________ $N$_________________

6. Give the coordinates of each vertex of figure $J'K'L'M'$.
   
   $J'$_________________ $K'$_________________ $L'$_________________ $M'$_________________ $N'$_________________

7. Complete this algebraic rule to describe the dilation.
   
   $(x, y) \rightarrow$___________________________

Li made a scale drawing of a room. The scale used was $5 \text{ cm} = 1 \text{ m}$.

The scale drawing is the preimage and the room is the dilated image.

8. What is the scale in terms of centimeters to centimeters?
   
   ______________________________________________________________________________________

9. Complete this algebraic rule to describe the dilation from the scale drawing to the room.
   
   $(x, y) \rightarrow$___________________________

10. The scale drawing measures 15 centimeters by 20 centimeters.
    What are the dimensions of the room?
    
    ______________________________________________________________________________________
Algebraic Representations of Dilations

Judy drew the gray square shown at the right. Then she drew an enlargement of that square and a reduction of that square.

1. Complete this algebraic rule to describe the enlargement.

\[(x, y) \rightarrow \text{___________________________}\]

2. Complete this algebraic rule to describe the reduction.

\[(x, y) \rightarrow \text{___________________________}\]

Roland made a scale drawing of a rectangular flower garden.
The scale used was 1 in. = \(\frac{3}{4}\) ft. The scale drawing is the preimage and the flower garden is the dilated image.

3. The coordinates of three of the vertices of the scale drawing are \((-3, 3), (2, 3),\) and \((2, -1)\). What are the coordinates of the fourth vertex?

_________________________________________________________________________________________

4. Complete this algebraic rule to describe the dilation from the scale drawing to the garden.

\[(x, y) \rightarrow \text{___________________________}\]

5. The scale drawing measures 5 in. by 4 in. What are the dimensions of the garden?

_________________________________________________________________________________________

6. Roland decides to make the dimensions of his garden 50% larger. What are the coordinates of the new scale drawing?

_________________________________________________________________________________________

7. What are the dimensions of the new scale drawing?

_________________________________________________________________________________________

8. What are the dimensions of the new garden?

_________________________________________________________________________________________

9. If Roland makes the dimensions of the original garden 50% smaller, what would the dimensions of the garden be?

_________________________________________________________________________________________
Algebraic Representations of Dilations

Use triangle $ABC$ for Exercises 1–4. The first one is done for you.

1. Give the coordinates of each vertex of $\triangle ABC$.
   
   $A(0, 6) \hspace{1cm} B(0, 0) \hspace{1cm} C(4, 0)$

2. Multiply each coordinate of the vertices of $\triangle ABC$ by 2 to find the vertices of the dilated image $\triangle A'B'C'$.
   
   $A'(\hspace{1cm}) \hspace{1cm} B'(\hspace{1cm}) \hspace{1cm} C'(\hspace{1cm})$

3. Graph $\triangle A'B'C'$.

4. Complete this algebraic rule to describe the dilation.
   
   $(x, y) \rightarrow \hspace{1cm}$

Use the figures at the right for Exercises 5–7. The first one is done for you.

5. Give the coordinates of each vertex of figure $JKLM$.
   
   $J(-2, -2) \hspace{1cm} K(-2, 2) \hspace{1cm} L(2, 2) \hspace{1cm} M(2, -2)$

6. Give the coordinates of each vertex of figure $J'K'L'M'$.
   
   $J'(\hspace{1cm}) \hspace{1cm} K'(\hspace{1cm}) \hspace{1cm} L'(\hspace{1cm}) \hspace{1cm} M'(\hspace{1cm})$

7. Complete this algebraic rule to describe the dilation.
   
   $(x, y) \rightarrow \hspace{1cm}$

Li made a scale drawing of a room. The scale used was 1 in. = 1 ft. The scale drawing is the pre-image and the room is the dilated image. The first one is done for you.

8. What is the scale in terms of inches to inches?
   
   1 inch = 12 inches

9. Complete this algebraic rule to describe the dilation from the scale drawing to the room.
   
   $(x, y) \rightarrow \hspace{1cm}$

10. The scale drawing measures 10 inches by 12 inches. What are the dimensions of the room?
You dilate a figure using the origin as the center of dilation. Multiply each coordinate by the scale factor. The scale factor is the number that describes the change in size in a dilation.

Using the origin O as the center of dilation, dilate \( \triangle ABC \) by a scale factor of 2.5.

\[
\begin{align*}
A(2, 2) &\rightarrow A'(2.5 \cdot 2, 2.5 \cdot 2) \text{ or } A'(5, 5) \\
B(4, 0) &\rightarrow B'(2.5 \cdot 4, 2.5 \cdot 0) \text{ or } B'(10, 0) \\
C(4, 2) &\rightarrow C'(2.5 \cdot 4, 2.5 \cdot 2) \text{ or } C'(10, 5)
\end{align*}
\]

Using the origin as the center of dilation, dilate \( \triangle ABC \) by a scale factor of 2. Graph the dilation.

1. \( A(1, 2) \rightarrow A'(2 \cdot 1, 2 \cdot 2) \text{ or } A'(\underline{2}, \underline{4}) \)
   
   \( B(2, 0) \rightarrow B'(\underline{4}, 0) \text{ or } B'(\underline{4}, \underline{0}) \)
   
   \( C(3, 3) \rightarrow C'(\underline{6}, \underline{6}) \text{ or } C'(\underline{6}, \underline{6}) \)

When the scale factor is a fraction between 0 and 1, the image is smaller than the original figure.

Using the origin O as the center of dilation, dilate \( \triangle ABC \) by a scale factor of \( \frac{1}{3} \).

\[
\begin{align*}
A(3, 3) &\rightarrow A'(\frac{1}{3} \cdot 3, \frac{1}{3} \cdot 3) \text{ or } A'(1, 1) \\
B(6, 0) &\rightarrow B'(\frac{1}{3} \cdot 6, \frac{1}{3} \cdot 0) \text{ or } B'(2, 0) \\
C(6, 6) &\rightarrow C'(\frac{1}{3} \cdot 6, \frac{1}{3} \cdot 6) \text{ or } C'(2, 2)
\end{align*}
\]

Using the origin as the center of dilation, dilate \( \triangle ABC \) by a scale factor of \( \frac{1}{2} \). Graph the dilation.

2. \( A(8, 0) \rightarrow A'(\frac{1}{2} \cdot 8, \frac{1}{2} \cdot 0) \text{ or } A'(\underline{4}, \underline{0}) \)
   
   \( B(4, 4) \rightarrow B'(\underline{2}, \underline{4}) \text{ or } B'(\underline{2}, \underline{4}) \)
   
   \( C(6, 8) \rightarrow C'(\underline{3}, \underline{4}) \text{ or } C'(\underline{3}, \underline{4}) \)
Algebraic Representations of Dilations

Reading Strategies: Build Vocabulary

A dilation changes the size of a figure without changing its shape. Some dilations are enlargements. Some dilations are reductions.

The gray figure is an enlargement.

The gray figure is a reduction.

The gray figures are called images of the black figure.

The black figures are the original figures.

Sometimes the original figures are called preimages.

Vertices of original figures or preimages are indicated with italic capital letters. For example, ABC.

Vertices of dilated figures or images are indicated with italic capital letters followed by a small mark called a prime symbol. For example, A'B'C'.

Complete.

1. The figures at the right show a reduction. Label the vertices of the original figure MNP. Label the vertices of the dilation M'N'P'.

2. Explain the difference between an enlargement and a reduction.
A dilation changes the size of a figure without changing its shape. Some dilations are enlargements. Some dilations are reductions.

**Problem 1**

What are the characteristics of an enlargement?

Look at the coordinates of corresponding vertices.

\[
\begin{align*}
A(0, -2) & \rightarrow A'(0, -4) & \text{The absolute values of the coordinates of } A', B', \text{ and } C' \text{ are greater than the absolute values of the coordinates of } A, B, \text{ and } C. \\
B(4, 4) & \rightarrow B'(8, 8) \\
C(4, -2) & \rightarrow C'(8, -4)
\end{align*}
\]

\[
\begin{align*}
\frac{A'C'}{AC} &= \frac{8}{4} = 2 \\
\frac{B'C'}{BC} &= \frac{12}{6} = 2
\end{align*}
\]

Find the ratios of corresponding lengths. The ratios are equal and are greater than 1.

\(A'B'C'\) is an enlargement of \(ABC\) because:

- The absolute values of the coordinates of the image are greater than the absolute values of the coordinates of the original figure.
- The ratio of corresponding lengths is greater than 1, and
- \(A'B'C'\) is the same shape as \(ABC\), but larger.

**Problem 2**

What are the characteristics of a reduction?

\(A'B'C'D'\) is a reduction of \(ABCD\) because:

- The absolute values of the coordinates of the image are less than the absolute values of the coordinates of the original figure.
- The ratio of corresponding lengths is less than 1, and
- \(A'B'C'D'\) is the same shape as \(ABCD\), but smaller.

Tell whether each dilation is an enlargement or a reduction.

1. 
2. 

Original content Copyright © by Houghton Mifflin Harcourt. Additions and changes to the original content are the responsibility of the instructor.
Identify a sequence of transformations that will transform figure A into figure C. Express each transformation algebraically.

1. What transformation is used to transform figure A to figure B?
   __________________________________________

2. What transformation is used to transform figure B to figure C?
   __________________________________________

3. Name two figures that are congruent. ________________

4. Name two figures that are similar, but not congruent. ________________

Complete each transformation.

5. Transform figure A to figure B by applying \((x, y) \rightarrow (2x, 2y)\).

6. Transform figure B to figure C by rotating it 90° clockwise around the origin.

7. Name two figures that are congruent. ________________

8. Name two figures that are similar, but not congruent. ________________

Geraldo designed a flag for his school. He started with \(\triangle ABC\). He used centimeter grid paper. To create the actual flag, the drawing must be dilated using a scale factor of 50. Express each transformation algebraically.

9. What transformation was used to create \(\triangle CBD\) from \(\triangle ABC\)?
   ______________________________________________________________

10. How long will each side of the actual flag \(ABD\) be?
    _____________________________________________________________

11. The principal decides he wants the flag to hang vertically with side \(AD\) on top. What transformation should Geraldo use on \(\triangle ABD\) on his drawing so it is in the desired orientation?
    ____________________________________________________________________
Similar Figures

Identify a sequence of transformations that will transform figure A into figure C. Express each transformation algebraically.

1. What transformation is used to transform figure A to figure B?

2. What two transformations are used to transform figure B to figure C?

3. Name two figures that are similar but not congruent.

Complete each transformation.

4. Transform figure A to figure B by applying

\[(x, y) \rightarrow \left( \frac{1}{3}x, \frac{1}{3}y \right).\]

5. Transform figure B to figure C with these two transformations:

\[(x, y) \rightarrow (-y, x)\]
\[(x, y) \rightarrow (x + 6, y)\]

6. If you performed the transformations of figure B to figure C in the opposite order, what would the coordinates of the right angle in figure C be?

Use the grid at the right for 7–9.

7. Draw any rectangle A of your choosing.

8. Dilate rectangle A. Label it B. Describe the transformation algebraically.

9. Rotate rectangle B about the origin. Label it C. Describe the transformation algebraically.
Identify a sequence of transformations that will transform figure $A$ into figure $C$. The first one is done for you.

1. What transformation is used to transform figure $A$ to figure $B$?

$(x, y) \rightarrow (2x, 2y)$

2. What transformation is used to transform figure $B$ to figure $C$?

3. Name two figures that are congruent.

4. Name two figures that are similar but not congruent.

Complete each transformation.

5. Rotate figure $A$ $180^\circ$ to figure $B$ by applying $(x, y) \rightarrow (-x, -y)$.

6. Transform figure $B$ to figure $C$ by dilating it by a factor of 2, so $(x, y) \rightarrow (2x, 2y)$.

7. Name two figures that are congruent.

8. Name two figures that are similar but not congruent.

Geraldo designed a flag for his school. He started with $\triangle ABC$. He used centimeter grid paper. Sides $AB$ and $BC$ are each 5 cm long.

9. How long is side $AC$ on Geraldo’s drawing?

10. To make the real flag, each side will be dilated by a factor of 100. How long will each side of the actual flag $ABC$ be?
Multiple dilations can be applied to a figure. If one of the transformations is a dilation, the figure and its image are similar. The size of the figure is changed but the shape is not.

In a dilation, when the scale is a greater than 1, the image is an enlargement. When the scale is a fraction between 0 and 1, the image is a reduction.

1st transformation: translation right 6 units
\((x, y) \rightarrow (x + 6, y)\),
relative size: congruent

2nd transformation: dilation by a scale of 3
\((x, y) \rightarrow (3x, 3y)\)
relative size: similar

The dilation at the right has a scale of \(\frac{1}{4}\).

Algebraically it is \((x, y) \rightarrow \left(\frac{1}{4}x, \frac{1}{4}y\right)\)

Describe each transformation. Express each one algebraically. Tell whether the figure and its image are congruent or are similar.

1. First transformation:
   Description: _________________________
   Algebraically: _________________________
   Relative size: _________________________

2. Second transformation:
   Description: _________________________
   Algebraically: _________________________
   Relative size: _________________________
Similar Figures

Reading Strategies: Use Logical Reasoning

Transformations that are dilations create similar figures.

Some dilations create enlargements.

Use logic to compare coordinates of vertices.

Black figure: (−3, 3), (−3, −2), (4, −2)
Gray figure: (−6, 6), (−6, −4), (8, −4)
Each coordinate of the gray figure is twice the coordinate of the black figure.
Algebraically: \((x, y) \rightarrow (2x, 2y)\)
This is an enlargement.

Some dilations create reductions.

Use logic to compare coordinates of vertices.

Black figure: (−4, −6), (0, 6), (8, −6)
Gray figure: (−2, −3), (0, 3), (4, −3)
Each coordinate of the gray figure is one half the coordinate of the black figure.
Algebraically: \((x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)\)
This is a reduction.

Use logic to compare coordinates. Describe the dilation algebraically.
Tell whether the dilation is an enlargement or a reduction.

1. Black figure: (6, 3), (−12, 9), (−12, 3)
   Algebraically: \((x, y) \rightarrow \) ____________
   Enlargement or reduction? ____________

2. Black figure: (5, 15), (−20, 15), (−10, 5)
   Algebraically: \((x, y) \rightarrow \) ____________
   Enlargement or reduction? ____________

3. Black figure: (1, 3), (−2, 1), (−2, 3)
   Algebraically: \((x, y) \rightarrow \) ____________
   Enlargement or reduction? ____________

   Gray figure: (8, 24), (−16, 8), (−16, 24)
Similar Figures

Success for English Learners

When including dilations in combining transformations, some transformations will produce an enlargement. Other combinations will produce a reduction. The original figure and its enlargement or reduction are similar, not congruent.

Problem 1
Perform the transformations in order.

\[(x, y) \rightarrow (-x, y)\]
\[(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)\]

Original figure: \((-2, 10), (-2, 0), (6, 10), (6, 0)\)
1st transformation: \((2, 10), (2, 0), (-6, 10), (-6, 0)\)
2nd transformation: \((1, 5), (1, 0), (-3, 5), (-3, 0)\)
The original figure is black.
The next figure is solid gray.
The final figure is dashed gray.
The final figure is smaller than the black figure.
The 2nd transformation produces a reduction.

Problem 2
Perform the transformations in order.

\[(x, y) \rightarrow (-y, x)\]
\[(x, y) \rightarrow (3x, 3y)\]

Original figure: \((0, 2), (0, 0), (3, 2), (3, 0)\)
1st transformation: \((-2, 0), (0, 0), (-2, 3), (0, 3)\)
2nd transformation: \((-6, 0), (0, 0), (-6, 9), (0, 9)\)
The original figure is black.
The next figure is solid gray.
The final figure is dashed gray.
The final figure is larger than the black figure.
The 2nd transformation produces an enlargement.

Complete.
1. Write a combination of transformations that will produce an enlargement.

2. Write a combination of transformations that will produce a reduction.
Transformations and Similarity

**Challenge**

You can combine transformations by performing a transformation on the original figure and then performing a second transformation on the image. The resulting image is called the final image. As with single transformations, the original figure and the final image are similar and may be congruent.

Treating each as a separate transformation, you would write:

\[(x, y) \rightarrow (2x, 2y)\] and \[(x, y) \rightarrow (−y, x).\]

Treating these as a combined transformation, you would write: \[(x, y) \rightarrow (−2y, 2x).\]

**For each sequence of transformations, write the algebraic representation of the combined transformations. Then write the letter of the final image that shows the transformations.**

1. \[(x, y) \rightarrow (−x, y)\] and \[(x, y) \rightarrow (x + 2, y)\]
   \[(x, y) \rightarrow \_\_\_\_\_\_\_\_\_\_; \text{ figure } \_\_\_\_\_\_\_\_\_\_\_\_\_\_.\]

2. \[(x, y) \rightarrow (−y, x)\] and \[(x, y) \rightarrow (x, y − 2)\]
   \[(x, y) \rightarrow \_\_\_\_\_\_\_\_\_\_; \text{ figure } \_\_\_\_\_\_\_\_\_.\]

3. \[(x, y) \rightarrow (x − 2, y)\] and \[(x, y) \rightarrow (−y, x)\]
   \[(x, y) \rightarrow \_\_\_\_\_\_\_\_\_\_; \text{ figure } \_\_\_\_\_\_\_\_\_.\]

4. \[(x, y) \rightarrow (−x, −y)\] and \[(x, y) \rightarrow (2x, 2y)\]
   \[(x, y) \rightarrow \_\_\_\_\_\_\_\_\_\_; \text{ figure } \_\_\_\_\_\_\_\_\_.\]

5. \[(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)\) and \[(x, y) \rightarrow (2x, 2y)\]
   \[(x, y) \rightarrow \_\_\_\_\_\_\_\_\_\_; \text{ figure } \_\_\_\_\_\_\_\_\_.\]

**Use your answers to Exercises 1–5 to answer each question.**

6. Does the order in which you perform combined transformations affect the final image? ______

7. Which of the final images are congruent to the original figure? ______
UNIT 4: Transformational Geometry

MODULE 9 Transformations and Congruence

LESSON 9-1

Practice and Problem Solving: A/B

1. 5 units right and 8 units down
2. 2 units left and 9 units up
3. 4.

5. a.

b. Area of $JKLM = 28$ square units, area of $JKLM' = 28$ square units

c. No; the image and preimage are congruent, so they have the same size. This means that the areas are the same.

Practice and Problem Solving: C

1. 2.

2.  
3.

a. \( A'(−6, 0), B'(−4, 2), C'(0, 2), D'(2, 0), E'(0, −2), F'(−4, −2) \)
b. \( A''(−6, −4), B''(−4, −2), C''(0, −2), D'(2, −4), E''(0, −6), F''(−4, −6) \)
c. \( A'''(0, −2), B'''(2, 0), C'''(6, 0), D'''(8, −2), E'''(6, −4), F'''(2, −4) \)
d. Yes; the figures cover the plane without overlapping and without any gaps.

4. \( P(−8, 6), Q(−5, 6), R(−6, 4) \)

**Practice and Problem Solving: D**

1. \( A(−7, −2) \)
2. \( B(6, 6) \)
3. \( C(−3, −5) \)
4. \( \text{side } AB' \)
5. \( \text{angle } C' \)
6. The translation moves the triangle 9 units left and 6 units down.
7. a. The point is translated 2 units right and 8 units down.
   
   b. \( P' \)

   c. They are congruent.

8.

9.

**Reteach**

1.

2.
3. Yes; translations preserve the size and shape of a figure. Even after two translations, the resulting figure is congruent to the original figure.

**Reading Strategies**
1. Triangle $A'B'C'$
2. Triangle $ABC$
3. 3 vertices
4. Yes; a translation produces a figure (image) that is congruent to the original figure (preimage).
5. The translation moves the triangle 5 units left and 7 units up.
6. A transformation is an operation that changes the position, size, or shape of a figure. A translation is a type of transformation that changes only the position of a figure.

**Success for English Learners**
1. The translation moved the triangle 5 units to the right.
2. Yes; the new translation is the same as the one in Problem 2, except that the vertical movement is described first and the horizontal movement is second.

**LESSON 9-2**

**Practice and Problem Solving: A/B**
1. Quadrilateral $G$
2. Quadrilaterals $F$ and $G$
3. One is a translation of the other.
4. 
5. 
6. a. 

**Practice and Problem Solving: C**
1. 
2. 
3. 
4. 
5. 
6. a. 

b. Perimeter of $KLMN = 12$ units, perimeter of $K'L'M'N' = 12$ units

c. No; the image and preimage are congruent, so they have the same size. This means that the perimeters are the same.
3. The $x$-coordinate for each point on the image is the opposite of the $x$-coordinate of the corresponding point on the figure. The $y$-coordinates stay the same.

4.

5. angle $D^\prime$
6. a reflection across the $y$-axis
7.

8.

9. flips
10. always
11. $y$-coordinate

**Reteach**

1.

**Practice and Problem Solving: D**

1. $A'(6, 2)$
2. $B(-5, 6)$
3. $C'(3, 7)$
4. side $C'D'$

Original content Copyright © by Houghton Mifflin Harcourt. Additions and changes to the original content are the responsibility of the instructor.
Reading Strategies
1. Triangle $C'D'E'$
2. Triangle $CDE$
3. Sample answer: $C$ and $C'$
4. a reflection across the $y$-axis
5. a. quadrilateral $PQRS$
   b. reflection
6. The corresponding points are the same distance from the line of reflection.

Success for English Learners
1. Reflection across the $y$-axis
2. Connect the reflected vertices to form triangle $A'B'C'$.

LESSON 9-3
Practice and Problem Solving: A/B
1. $B$
2. $C$
3. $B$
4. $D$
5. 30 cm, 40 cm, and 50 cm
6. III
7. I
8. IV
9. II
10. 60° and 120°

Practice and Problem Solving: C
1. Sample answer: Not a rotation because triangle $B$ is flipped from where it would be after a rotation.
2. A rotation of 180°
3. A rotation of 90° counterclockwise OR 270° clockwise
4. a regular hexagon
5. A large square is formed with its center at the origin and each side is twice as long as the side of square $S$. 
6. 
7. 
12. Accept: reflection over $x$-axis, translation of 5 units down, or rotation of 270° counterclockwise.
Practice and Problem Solving: D
1. B
2. C
3. B
4. D
5. 2 cm and 4 cm
6. I
7. I
8. III
9. II
10. 

11. The image will be the same as triangle K.

Reteach
1. D
2. B
3. C
4. B
5. 3 cm, 4 cm, 5 cm
6. Sample answer: A rotation of 180° turns the figure a half-turn and will be the same whether turned clockwise or counterclockwise.

Reading Strategies
1. Check student’s answers. Sample answer: One side will go from the x-axis to the y-axis maintaining a length of 4. Vertex at (−3, 4) will go to (4, 3)

Success for English Learners
1. 90° counterclockwise or 270° clockwise
2. 90° clockwise or 270° counterclockwise

LESSON 9-4
Practice and Problem Solving: A/B
1. \((x, y) \rightarrow (x, y - 5)\); translation down 5 units
2. \((x, y) \rightarrow (-y, x)\); rotation 90° counterclockwise
3. reflection over the y-axis
4. rotation of 180°
5. \(A'(2, 1), B'(-3, 2), C'(-1, 6)\)
6. a 90° clockwise rotation
Practice and Problem Solving: C
1. \((x, y) \rightarrow (x + 2, y);\) translation right 2 units
2. \((x, y) \rightarrow (-y, x);\) rotation 90º counterclockwise
3. reflection over the \(x\)-axis
4. rotation of 180º
5. Possible answer: \((x, y) \rightarrow (x + 2, y)\)
6. \((0, -3);\) rotation of 90º clockwise

Practice and Problem Solving: D
1. \((x, y) \rightarrow (-x, -y);\) rotation 180º clockwise
   OR counterclockwise
2. \((x, y) \rightarrow (x, y + 5);\) translation up 5 units
3. \((x, y) \rightarrow (-x, y);\) reflection over the \(y\)-axis
4. \((x, y) \rightarrow (y, -x);\) rotation 90º clockwise
5. \((-4, 4)\)
6. \((-1, 4)\)
7. \((-2, 1)\)

Reteach
1. reflection over the \(y\)-axis
2. 90º rotation counterclockwise
3. translation up 4 units
4. 180º rotation
5. reflection over the \(x\)-axis

Reading Strategies
1. translation up 2 units
2. 90º rotation clockwise
3. 180º rotation
4. reflection over the \(y\)-axis

Success for English Learners
1. reflection or rotation
2. translation
3. rotation
4. rotation 90º clockwise
5. translation right 2 units
6. reflection over \(x\)-axis

LESSON 9-5
Practice and Problem Solving: A/B
1. rotation 90º counterclockwise
2. translation right 4 units
3. \((x, y) \rightarrow (-y, x);\) \((x, y) \rightarrow (x + 4, y)\)
4–6.

7. yes
8. different
9. Sample answer: rotation 90° clockwise, translation 4 units left
10. size: no; orientation: yes

Practice and Problem Solving: C
1. Sample answer: \((x, y) \rightarrow (-x, -y); (x, y) \rightarrow (x, y + 4)\)
2. Sample answer: \((x, y) \rightarrow (-y, x); (x, y) \rightarrow (x + 2, y)\)
3. yes
4. square; Sample explanation: Each side of the square is a hypotenuse of congruent triangles. Each angle is supplemental to two angles with a sum of 90°.
5. Sample answer: \((x, y) \rightarrow (y, -x); (x, y) \rightarrow (x - 4, y)\)
6. Accept answers that meet the criteria.
   Sample transformation of \(A\): \((x, y) \rightarrow (x - 5, y)\)

Practice and Problem Solving: D
1. rotation 90° counterclockwise
2. translation down 6 units
3. \((x, y) \rightarrow (-y, x); (x, y) \rightarrow (x, y - 6)\)

4. 7, and 10
5. yes
6. different
7. yes
8. yes
9. same
11. yes
12. different
13. yes
14. different

Reteach
1. reflection over the \(y\)-axis; \((x, y) \rightarrow (-x, y); different\)
2. 90° rotation counterclockwise; \((x, y) \rightarrow (-y, x); different\)

Reading Strategies
1. translation down 4 units
2. \(y\) decreases by 4
3. \((x, y - 4)\)
4. yes
5. 180° rotation
6. \(x\) and \(y\) both change signs.
7. \((-x, -y)\)
8. yes

Success for English Learners
1. translation; \((x, y) \rightarrow (x + 7, y - 6)\)
2. reflection; \((x, y) \rightarrow (x, -y)\)
3. rotation; \((x, y) \rightarrow (-x, -y)\)
MODULE 9 Challenge
1. \((x, y) \rightarrow (x + h, y + k)\); translation, direct
2. \((x, y) \rightarrow (x - y)\); reflection across \(x\)-axis, opposite
3. \((x, y) \rightarrow (-y, x)\); rotation of 90°, opposite
4. \((x, y) \rightarrow (-x, y)\); reflection across \(y\)-axis, opposite
5. translation
6. rotation, reflection

MODULE 10 Transformations and Similarity

LESSON 10-1

Practice and Problem Solving: A/B
1. 2, 2; 6, 6
2. \(\frac{6}{2} = 3\); \(\frac{6}{2} = 3\)
3. Yes
4. enlargement
5. No, the ratios are not all equal.
   \[
   \frac{3}{12} = \frac{1}{4}; \frac{4}{16} = \frac{1}{4}; \frac{5}{25} = \frac{1}{5}
   \]
6. Yes, this shows a reduction. The ratio of the lengths of corresponding sides is \(\frac{1}{2}\).
7. Yes, this shows an enlargement. The ratio of the lengths of corresponding sides is \(\frac{3}{1}\).
8. Yes; The lines drawn through corresponding vertices meet in a single point.

Practice and Problem Solving: C
1. 2.5
2. \(\frac{1}{3}\)

Practice and Problem Solving: D
1. 3; 2; 9; 6
2. \(\frac{9}{3} = 3\); \(\frac{6}{2} = 3\)
3. Yes
4. Enlargement
5. 6, 6, 6, 6; 3, 3, 3, 3
6. \(\frac{3}{6} = \frac{1}{2}; \frac{3}{6} = \frac{1}{2}; \frac{3}{6} = \frac{1}{2}; \frac{3}{6} = \frac{1}{2}\)
7. Yes
8. Reduction
9. Enlargement

Reteach
1. \(\frac{4}{3} = \frac{1}{3}; \frac{3}{4} = \frac{3}{4}\); no; no
2. \(\frac{2}{4} = \frac{1}{2}; \frac{4}{8} = \frac{1}{2}\); yes; yes
Reading Strategies
1. no
2. yes

Success for English Learners
1. enlargement
2. reduction

LESSON 10-2

Practice and Problem Solving: A/B
1. \(A(0, 4), B(0, 0), C(5, 0)\)
2. \(A'(0, 8), B'(0, 0), C'(10, 0)\)
3. 

```
\[\begin{array}{c|c|c|c|c}
& \text{x} & \text{y} \\
\hline
\text{A} & 0 & 4 \\
\text{B} & 0 & 0 \\
\text{C} & 5 & 0 \\
\end{array}\]
```

4. \((2x, 2y)\)
5. \(J(-2, -2), K(-2, 2), L(0, 2), M(1, 0), N(0, -2)\)
6. \(J'(-6, -6), K'(-6, 6), L'(0, 6), M'(3, 0), N'(0, -6)\)
7. \((3x, 3y)\)
8. 1 cm = 20 cm
9. \((20x, 20y)\)
10. 300 cm by 400 cm or 3 m by 4 m

Practice and Problem Solving: C
1. \(\left(\frac{1}{2}x, \frac{1}{2}y\right)\)
2. \(\left(\frac{1}{4}x, \frac{1}{4}y\right)\)
3. \((-3, -1)\)
4. \((9x, 9y)\)
5. 45 in. by 36 in.
6. \((-4\frac{1}{2}, 4\frac{1}{2}), \left(3, 4\frac{1}{2}\right), \left(3, -1\frac{1}{2}\right), \left(-4\frac{1}{2}, -1\frac{1}{2}\right)\)

Practice and Problem Solving: D
1. \(A(0, 6), B(0, 0), C(4, 0)\)
2. \(A'(0, 12), B'(0, 0), C'(8, 0)\)
3. 

```
\[\begin{array}{c|c|c|c|c}
& \text{x} & \text{y} \\
\hline
\text{A} & 0 & 6 \\
\text{B} & 0 & 0 \\
\text{C} & 4 & 0 \\
\end{array}\]
```

4. \((2x, 2y)\)
5. \(J(-2, -2), K(-2, 2), L(2, 2), M(2, -2)\)
6. \(J'(-6, -6), K'(-6, 6), L'(6, 6), M'(6, -6)\)
7. \((3x, 3y)\)
8. 1 in. = 12 in.
9. \((12x, 12y)\)
10. 120 in. by 144 in. or 10 ft by 12 ft

Reteach
1. \(A(1, 2) \rightarrow A'(2\cdot1, 2\cdot2)\) or \(A'(2, 4)\)
2. \(B(2, 0) \rightarrow B'(2\cdot2, 2\cdot0)\) or \(B'(4, 0)\)
3. \(C(3, 3) \rightarrow C'(2\cdot3, 2\cdot3)\) or \(C'(6, 6)\)

Original content Copyright © by Houghton Mifflin Harcourt. Additions and changes to the original content are the responsibility of the instructor.
2. $A(8, 0) \rightarrow A'(\frac{1}{2} \cdot 8, \frac{1}{2} \cdot 0)$ or $A'(4, 0)$

$B(4, 4) \rightarrow B'(\frac{1}{2} \cdot 4, \frac{1}{2} \cdot 4)$ or $B'(2, 2)$

$C(6, 8) \rightarrow C'(\frac{1}{2} \cdot 6, \frac{1}{2} \cdot 8)$ or $C'(3, 4)$

Reading Strategies
1. Sample answer:

Success for English Learners
1. reduction
2. enlargement

LESSON 10-3

Practice and Problem Solving: A/B
1. $(x, y) \rightarrow (2x, 2y)$
2. $(x, y) \rightarrow (x - 4, y)$
3. Figures B and C
4. Figures A and B or Figures A and C

5–6.

7. Figures B and C
8. Figures A and B or Figures A and C
9. $(x, y) \rightarrow (x, -y)$
10. $AB = 250 \text{ cm}; BD = 250 \text{ cm}; AD = 300 \text{ cm}$
11. $(x, y) \rightarrow (y, -x)$

Practice and Problem Solving: C
1. $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$

2. Sample answer: $(x, y) \rightarrow (x, -y);$ $(x, y) \rightarrow (x + 5, y)$
3. Figures A and B or Figures A and C

4–5.

6. $(3, 4)$
7. Check students’ work. Sample answer:

8. Sample answer: \((x, y) \rightarrow (3x, 3y)\)

9. Sample answer: \((x, y) \rightarrow (-x, y)\)

Practice and Problem Solving: D

1. dilating by a factor of 2 OR \((x, y) \rightarrow (2x, 2y)\)

2. translating down 6 units OR \((x, y) \rightarrow (x, y - 6)\)

3. Figures B and C

4. Figures A and B or Figures A and C

5–6.

7. Figures A and B

8. Figures A and C or Figures B and C

9. 6 cm

10. \(AB = 500\) cm; \(BC = 500\) cm; \(AC = 600\) cm

Reteach

1. dilation by scale of \(\frac{1}{3}\); \((x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)\); similar

2. translation down 7 units; \((x, y) \rightarrow (x, y - 7)\); congruent

Reading Strategies

1. \((x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)\); reduction

2. \((x, y) \rightarrow \left(\frac{1}{5}x, \frac{1}{5}y\right)\); reduction

3. \((x, y) \rightarrow (8x, 8y)\); enlargement

Success for English Learners

1. Check students’ work. It should include a dilation with a whole number greater than 1.

2. Check students’ work. It should include a dilation with a fraction between 0 and 1.

MODULE 10 Challenge

1. \((-x + 2, y)\); \(L\)

2. \((-y, x - 2)\); \(J\)

3. \((-y, x - 2)\); \(J\)

4. \((-2x, -2y)\); \(M\)

5. \((x, y)\); \(K\)

6. yes

7. \(J, K, L\)