Parallel Lines Cut by a Transversal

Practice and Problem Solving: A/B

Use the figure at the right for Exercises 1–6.
1. Name both pairs of alternate interior angles.

2. Name the corresponding angle to \( \angle 3 \).

3. Name the relationship between \( \angle 1 \) and \( \angle 5 \).

4. Name the relationship between \( \angle 2 \) and \( \angle 3 \).

5. Name an interior angle that is supplementary to \( \angle 7 \).

6. Name an exterior angle that is supplementary to \( \angle 5 \).

Use the figure at the right for problems 7–10. Line \( MP \parallel line QS \).
Find the angle measures.
7. \( m\angle KRQ \) when \( m\angle KNM = 146^\circ \)

8. \( m\angle QRN \) when \( m\angle MNR = 52^\circ \)

If \( m\angle RNP = (8x + 63)^\circ \) and \( m\angle NRS = 5x^\circ \), find the following angle measures.

9. \( m\angle RNP = \) \hspace{1cm} 10. \( m\angle NRS = \)

In the figure at the right, there are no parallel lines. Use the figure for problems 11–14.
11. Name both pairs of alternate exterior angles.

12. Name the corresponding angle to \( \angle 4 \).

13. Name the relationship between \( \angle 3 \) and \( \angle 6 \).

14. Are there any supplementary angles? If so, name two pairs. If not, explain why not.
Parallel Lines Cut by a Transversal

Practice and Problem Solving: C

At the right is a map of Littleton. North Street, Center Street, and South Street are parallel.
Use the description to complete Exercises 1–2.

1. Avenue A is perpendicular to North Street. What is the relationship between Avenue A and South Street? Explain your reasoning.

2. Avenue B makes a 70° angle with Center Street. What are the measures of the angles that South Street makes with Avenue B?

In the figure at the right line \( BD \parallel \) line \( EG \).

3. If \( m\angle ACB = 5x° \), what is \( m\angle ACD \) in terms of \( x \)?

4. If \( m\angle BCF = 4x° \) and \( m\angle CFE = 11x° \), what is \( m\angle BCF \) and \( m\angle CFE \) in degrees?

5. If \( m\angle CFG = 3x° \) and \( m\angle DCF = (7x + 40)° \), what is \( m\angle CFG \) and \( m\angle DCF \) in degrees?

Use the figure at the right for Exercises 6–8.

6. Label the parallel lines and the transversal with letters.

7. Name all the angles that are congruent to the smaller angle.

8. Write a problem about the figure.
Parallel Lines Cut by a Transversal

Practice and Problem Solving: D

Use the figure at the right for Exercises 1–6. The first one is done for you.

1. What do the arrows that are in the middle of lines \(a\) and \(b\) mean? 
   
   The lines are parallel.

2. Name two parallel lines. ___________________________

3. Name the transversal. ___________________________

4. Name a pair of alternate exterior angles. ________________

5. Name an angle that is congruent to \(\angle 2\). ________________

6. Name an angle that is supplementary to \(\angle 2\). ________________

Line \(MP \parallel \text{ line } QS\). Find the angle measure.
The first one is done for you.

7. \(m\angle TRS\) when \(m\angle TNP = 40^\circ\) ______

8. \(m\angle QRU\) when \(m\angle MNR = 30^\circ\) ______

9. \(m\angle MNR\) when \(m\angle QRT = 145^\circ\) ______

10. \(m\angle PNU\) when \(m\angle SRU = 130^\circ\) ______

11. In the space below, draw two lines that are NOT parallel. Label them line \(g\) and line \(h\). Then draw a transversal and label it line \(k\).
Parallel Lines Cut by a Transversal

Parallel Lines

Parallel lines never meet.

Parallel Lines Cut by a Transversal

A line that crosses parallel lines is a **transversal**.

Eight angles are formed. If the transversal is not perpendicular to the parallel lines, then four angles are acute and four are obtuse.

The acute angles are all congruent.

The obtuse angles are all congruent.

Any acute angle is supplementary to any obtuse angle.

In each diagram, parallel lines are cut by a transversal. Name the angles that are congruent to the indicated angle.

1. The angles congruent to \( \angle 1 \) are: __________________

2. The angles congruent to \( \angle a \) are: ________________

3. The angles congruent to \( \angle z \) are: ________________

In each diagram, parallel lines are cut by a transversal and the measure of one angle is given. Write the measures of the remaining angles on the diagram.

4. \( 150^\circ \)

5. \( 135^\circ \)

6. \( 25^\circ \)
Parallel Lines Cut by a Transversal

Reading Strategies: Build Vocabulary

When parallel lines are cut by a third line, called a transversal, some pairs of angles are congruent. Congruent angles have the same measure.

Alternate angles lie on opposite sides of the transversal.

Adjacent angles share a common side.

Alternate interior angles are not adjacent angles and are between the parallel lines.

∠3 and ∠5 are one pair of alternate interior angles.
∠4 and ∠6 are another pair of alternate interior angles.

Alternate exterior angles are not adjacent angles and are outside the parallel lines.

∠1 and ∠7 are one pair of alternate exterior angles.
∠2 and ∠8 are another pair of alternate exterior angles.

Corresponding angles are angles that lay on the same side of the transversal and on the same side of the two parallel lines.

∠1 and ∠5 are one pair of corresponding angles.
∠2 and ∠6 are a second pair of corresponding angles.
∠3 and ∠7 are a third pair of corresponding angles.
∠4 and ∠8 are a fourth pair of corresponding angles.

Label one pair of angles with 1 and 2 for each type of angle pairs.

1. Corresponding angles

2. Alternate interior angles

3. Alternate exterior angles
Problem 1
Below are pairs of corresponding angles. When two of the lines are parallel, the corresponding angles are congruent. Congruent angles have the same measure.

Problem 2
Below are pairs of alternate interior angles. When two lines are parallel, the alternate interior angles are congruent. Below are pairs of alternate exterior angles. When two lines are parallel, the alternate exterior angles are congruent.

Use the figure at the right to answer the questions below.

1. Identify all the pairs of corresponding angles.

________________________________________

________________________________________

2. Identify all the pairs of alternate interior angles.

________________________________________

3. Identify all the pairs of alternate exterior angles.

________________________________________
LESSON 11-2

Angle Theorems for Triangles

Practice and Problem Solving: A/B

Find the unknown angle measure in each triangle. Choose the letter for the best answer.

1. \[
\begin{array}{c}
45^\circ \\
90^\circ \\
? \\
\end{array}
\]
   A) 45°  
   B) 55°  
   C) 90°  
   D) 135°

2. \[
\begin{array}{c}
60^\circ \\
? \\
70^\circ \\
\end{array}
\]
   A) 40°  
   B) 50°  
   C) 60°  
   D) 70°

Find the unknown angle measure in each triangle.

3. \[
\begin{array}{c}
85^\circ \\
65^\circ \\
? \\
\end{array}
\]

4. \[
\begin{array}{c}
104^\circ \\
? \\
30^\circ \\
\end{array}
\]

5. \[
\begin{array}{c}
85^\circ \\
40^\circ \\
? \\
\end{array}
\]

Find the value of the variable in problems 6–8.

6. \[
\begin{array}{c}
x \\
70^\circ \\
70^\circ \\
\end{array}
\]

7. \[
\begin{array}{c}
y \\
55^\circ \\
60^\circ \\
\end{array}
\]

8. \[
\begin{array}{c}
n^\circ \\
50^\circ \\
? \\
\end{array}
\]

Use the diagram at the right to answer each question below.

9. What is the measure of \( \angle DEF \)?

10. What is the measure of \( \angle DEG \)?

11. A triangular sign has three angles that all have the same measure. What is the measure of each angle?
Angle Theorems for Triangles

Practice and Problem Solving: C

Find the value of each variable.

1. \((x + 3)^\circ\)
2. \((n - 2)^\circ\)
3. \(t^\circ\)

4. \(w^\circ\)
5. \((4w - 6)^\circ\)
6. \(2x^\circ\)

Write and solve an equation to find the measures of the angles of each triangle.

7. The measure of each of the base angles of an isosceles triangle is 9° less than 4 times the measure of the vertex angle.

\[
\text{measure of each base angle} = \_
\]
\[
\text{measure of vertex angle} = \_
\]

8. The measure of the vertex angle of an isosceles triangle is one-fourth that of a base angle.

\[
\text{measure of each base angle} = \_
\]
\[
\text{measure of vertex angle} = \_
\]

9. The second angle in a triangle is twice as large as the first. The third angle is three times as large as the first. Find the angle measures.
Lesson 11-2

Angle Theorems for Triangles

Practice and Problem Solving: D

Find the measure of each unknown angle. The first one is done for you.

1. \[ \begin{align*}
85° + 40° & = 125° \\
180° - 125° & = 55° 
\end{align*} \]

2. \[ 14° + \ ? = 125° \]

3. \[ \begin{align*}
55° + 51° & = 106° \\
180° - 106° & = 74° 
\end{align*} \]

4. \[ \begin{align*}
23° + 141° & = 164° \\
180° - 164° & = 16° 
\end{align*} \]

5. \[ 64° + 76° + \ ? = 180° \]

6. \[ \begin{align*}
32° + 36° & = 68° \\
180° - 68° & = 112° 
\end{align*} \]

7. \[ \begin{align*}
47° + \ ? & = 120° \\
180° - 120° & = 60° 
\end{align*} \]

8. \[ \begin{align*}
57° + 55° & = 112° \\
180° - 112° & = 68° 
\end{align*} \]

9. \[ \begin{align*}
45° + 53° & = 98° \\
180° - 98° & = 82° 
\end{align*} \]

Find the value of each variable. The first one is done for you.

10. \[ \begin{align*}
55° + 60° & = 115° \\
180° - 115° & = 65° 
\end{align*} \]

11. \[ \begin{align*}
? & = 180° - 50° 
\end{align*} \]

12. \[ \begin{align*}
? & = 180° - 110° - 40° 
\end{align*} \]
If you know the measure of two angles in a triangle, you can subtract their sum from 180°. The difference is the measure of the third angle.

The two known angles are 60° and 55°.

\[
60° + 55° = 115° \\
180° - 115° = 65°
\]

Solve.

1. Find the measure of the unknown angle.

   Add the two known angles: ____ + ____ = ____

   Subtract the sum from 180°: 180° - ____ = ____

   The measure of the unknown angle is: ___

2. Find the measure of the unknown angle.

   Add the two known angles: ____ + ____ = ____

   Subtract the sum from 180°: 180° - ____ = ____

   The measure of the unknown angle is: ___

\angle DEG is an exterior angle.

The measure of \angle DEG is equal to the sum of \angle D and \angle F.

\[
47° + 30° = 77°
\]

You can find the measure of \angle DEF by subtracting 77° from 180°.

\[
180° - 77° = 103°
\]

The measure of \angle DEF is 103°.

Solve.

3. Find the measure of angle y.

   \[
65° + 85° = \_
\]

4. Find the measure of angle x.

   \[
180° - ___ = ___
\]
An **interior angle** of a triangle is an angle that is inside the triangle, and is formed by two sides of the triangle. Angles $A$, $B$, and $C$ are the interior angles.

The three **interior angles** of a triangle always have a sum of $180^\circ$.

Write an equation to find an unknown interior angle in a triangle.

\[ m\angle A + m\angle B + m\angle C = 180^\circ \]

**Find the measure of each unknown angle.**

1. 
2. 

\[ 60^\circ, 80^\circ, x^\circ \]

\[ (2x - 5)^\circ, x^\circ, (x + 25)^\circ \]

An **exterior angle** is an angle on the outside of a triangle, formed by two of the sides of a triangle. $\angle D$ is formed by $BC$ and $AC$.

The **remote interior angles** are the two interior angles that are not next to the exterior angle.

In this triangle, $\angle A$ and $\angle B$ are the remote interior angles to $\angle D$.

The sum of the measures of the two remote interior angles is equal to the measure of the exterior angle.

You can find an unknown exterior angle in a triangle by adding the measures of the remote interior angles.

\[ m\angle A + m\angle B = m\angle D \]

**Solve.**

3. Find the measure of the exterior angle.
Problem 1

What I know:
- One angle measures $37^\circ$.
- The sum of the angle measures has to equal $180^\circ$.

What I need:
- Another angle measure

Now I can find $y^\circ$.

\[
37^\circ + 90^\circ + y^\circ = 180^\circ \\
127^\circ + y^\circ = 180^\circ \\
y^\circ = 53^\circ
\]

Problem 2

What is $m$?

Remember:
All equilateral triangles have angle measures of $60^\circ$.

Find the measure of each unknown angle in the triangles shown below.

1. 

2.
Angle-Angle Similarity

Practice and Problem Solving: A/B

Explain whether the triangles are similar.

1.  

2.  

The diagram below shows a Howe roof truss, which is used to frame the roof of a building. Use it to answer problems 3–5.

3. Explain why \( \triangle LQN \) is similar to \( \triangle MPN \).

4. What is the length of support \( MP \)? ______________

5. Using the information in the diagram, can you determine whether \( \triangle LQJ \) is similar to \( \triangle KRJ \)? Explain.

6. In the diagram at the right, sides \( SV \) and \( RW \) are parallel. Explain why \( \triangle RTW \) is similar to \( \triangle STV \).
LESSON 11-3

Angle-Angle Similarity

Practice and Problem Solving: C

Solve.

1. Which of the triangles at the right are similar? Explain why.
   ___________________________________
   ___________________________________
   ___________________________________

2. Can you assume that the similar triangles in problem 1 are congruent? Explain.
   ___________________________________
   ___________________________________
   ___________________________________

Find the unknown measure.

3. A tree casts a shadow 21 feet long, while Enzo, who is 5 feet tall, casts a shadow 6 feet long.
   Height of tree = ________________

4. A zip line starts 35 feet above the ground and has one landing platform. The line is secured into the ground.
   Height of platform = ________________

In the diagram, segments BC and GH are parallel.
Use the diagram for Exercises 5–6.

5. Explain why \( \triangle ABC \) is similar to \( \triangle FGH \).
   ___________________________________
   ___________________________________
   ___________________________________

6. The triangles have the following coordinates:
   \( A(2, 4), B(2, 10), C(10, 10), F(14, 13), G(14, 16) \). Find the coordinates of \( H \).
   ___________________________________
   ___________________________________
Find the missing angle measure in each triangle. The first one is done for you.

1. 2. 3. 4.

5. Which two triangles in problems 1–4 are similar? Explain.

A cactus casts a shadow 33 feet long. At the same time of day, Liam, who is 6 feet tall, casts a shadow 9 feet long, as shown.

6. The triangles are redrawn below. Write the given information on the two new triangles.

7. Explain how you know that the two triangles are similar.

8. Complete and solve the proportion below to find the height of the cactus.

\[
\frac{6}{x} = \frac{24}{9} \\
x = \text{____ ft}
\]

Answer the questions about the diagram, which shows two parallel lines cut by a transversal.

9. What are the measures of $\angle RST$ and $\angle VWT$? Explain how you found the angle measures.

10. Explain whether $\triangle RST$ and $\triangle WVT$ are similar.

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**Angle-Angle Similarity**

**Reteach**

When solving triangle similarity problems involving proportions, you can use a table to organize given information and set up a proportion.

A telephone pole casts the shadow shown on the diagram. At the same time of day, Sandy, who is 5 feet tall, casts a shadow 8 feet long, as shown. Find the height of the telephone pole.

Organize distances in a table. Then use the table to write a proportion.

<table>
<thead>
<tr>
<th>Pole</th>
<th>Sandy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (ft)</td>
<td>x</td>
</tr>
<tr>
<td>Length of shadow (ft)</td>
<td>24 + 8, or 32</td>
</tr>
</tbody>
</table>

Solve the proportion. The height of the telephone pole is 20 feet.

Complete the table. Then find the unknown distance.

1. A street lamp casts a shadow 31.5 feet long, while an 8-foot tall street sign casts a shadow 14 feet long.
2. A 5.5-foot woman casts a shadow that is 3 feet longer than her son’s shadow. The son casts a shadow 13.5 feet long.

<table>
<thead>
<tr>
<th>Lamp</th>
<th>Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (ft)</td>
<td></td>
</tr>
<tr>
<td>Length of shadow (ft)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Woman</th>
<th>Son</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (ft)</td>
<td></td>
</tr>
<tr>
<td>Length of shadow (ft)</td>
<td></td>
</tr>
</tbody>
</table>

Height of street lamp = ____________

Height of son = ____________
Angle-Angle Similarity

Reading Strategies: Analyze Diagrams

For complicated diagrams that involve overlapping triangles, you can redraw the triangles so that they do not overlap. Transfer the information given in the original diagram to the new triangles.

The diagram shows the shadows cast by a tree and by Zachary at the same time of day.

Solve.

1. What does $ED$ represent? ___________________________________

2. What does $AB$ represent? ___________________________________

3. Segment $AE$ is not shown when the triangles are redrawn. What does $AE$ represent?

_________________________________________________________________________________________

4. Explain why $\triangle ABC$ is similar to $\triangle ECD$.

_________________________________________________________________________________________

5. Complete the statements about the similar triangles.

a. Side $AD$ corresponds to side ________________________________

b. Side $CE$ corresponds to side ________________________________

6. Write and solve a proportion to find the height of the tree.

_________________________________________________________________________________________
Problem 1
What are similar triangles?

Similar triangles are shown.

- Corresponding angles are congruent:
  \[ \angle A \cong \angle J \quad \angle B \cong \angle K \quad \angle C \cong \angle L \]

- Corresponding sides are proportional:
  \[ \frac{AB}{JK} = \frac{BC}{KL} = \frac{AC}{JL} \]

Problem 2
Explain whether the triangles are similar.

1. Describe the relationship between the angles of similar triangles and the sides of similar triangles.

2. Explain how the measures of the angles of two triangles are used to show that the triangles are similar.

Does given information show two pairs of angles congruent?

No

Find measures of third angles.

Are two pairs of angles congruent?

Yes

The triangles are similar.
Angle Relationships in Parallel Lines and Triangles

**Challenge**

1. Use what you know about exterior angles and remote interior angles to find the measure of $\angle x$. Show your work.

   $\angle x = 58\degree$

   $\angle x = (7n + 2)\degree$

   $\angle x = (9n - 8)\degree$

   Use your observation about the alternate interior angles of parallel lines to find the measure of $\angle x$ in each of these diagrams. Explain your reasoning.

2. $\angle x = 48\degree$

3. $\angle x = 60\degree$

4. $\angle x = 70\degree$

5. $\angle x = 35\degree$

6. $\angle x = 45\degree$

7. $\angle x = 30\degree$
Find the missing side to the nearest tenth.

1. \( \triangle ABC \) with sides 5, 4, and \( c \)
2. \( \triangle ABC \) with sides 15, 25, and \( b \)
3. \( \triangle ABC \) with sides 48, \( a \), and 60

4. \( \triangle ABC \) with sides 23, 14, and \( b \)
5. \( \triangle ABC \) with sides 29, \( a \), and 23
6. \( \triangle ABC \) with sides 78, \( b \), and 30

Solve.

7. Jane and Miguel are siblings. They go to different schools. Jane walks 6 blocks east from home. Miguel walks 8 blocks north. How many blocks apart would the two schools be if you could walk straight from one school to the other?

________________________________________________________________________________________

8. The base of a rectangular box has a width of 3 inches and a length of 4 inches. The box is 12 inches tall.
   a. Draw a picture of the box below.

   b. How far is it from one of the box’s top corners to the opposite corner of the base of the box?

_____________________________________________________________________________________

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The Pythagorean Theorem

Practice and Problem Solving: C

In an isosceles right triangle the lengths of the sides $\overline{AC}$ and $\overline{BC}$ are equal.

Find the length of the hypotenuse to the nearest tenth of a unit in these isosceles right triangles with the given leg lengths.

1. $AC = BC = 1$ inch
2. $AC = BC = 2.5$ kilometers
3. $AC = BC = \sqrt{3}$ feet

In the isosceles triangle, find the length of $\overline{AB}$ when the lengths of $\overline{AC}$ and $\overline{CD}$ are given.

4. $AC = 4$, $CD = 2$
5. $AC = 10$, $CD = 6$
6. $AC = \sqrt{3}$, $CD = 1$

7. A diagonal of a cube goes from one of the cube’s top corners to the opposite corner of the base of the cube. Find the length of a diagonal $d$ in a cube that has an edge of length 10 meters.
The Pythagorean Theorem

Practice and Problem Solving: D

Find the length of the hypotenuse in each triangle using the Pythagorean Theorem, \( a^2 + b^2 = c^2 \). The first one is done for you.

1. \( R \)

\[
\begin{align*}
12^2 + 9^2 &= c^2 \\
&= c^2 \\
&= 15
\end{align*}
\]

2. \( X \)

\[
\begin{align*}
10^2 + 24^2 &= c^2 \\
&= c^2 \\
&= 670
\end{align*}
\]

3. \( A \)

\[
\begin{align*}
10^2 + 7.5^2 &= c^2 \\
&= c^2 \\
&= 132.5
\end{align*}
\]

Write the correct answer. Round your answers to the nearest tenth. The first one is done for you.

4. A right triangle has legs that are 10 meters and 3 meters in length. How long is its hypotenuse?

**Solution:**

\[
\begin{align*}
c^2 &= 10^2 + 3^2 \\
c^2 &= 100 + 9 \\
c^2 &= 109 \\
c &\approx 10.4 \text{ meters}
\end{align*}
\]

5. A soccer field is 120 yards long and 60 yards wide. How long is the diagonal of the field?

**Solution:**

\[
\begin{align*}
c^2 &= 120^2 + 60^2 \\
c^2 &= 14400 + 3600 \\
c^2 &= 18000 \\
c &\approx 134.2 \text{ yards}
\end{align*}
\]

Solve.

6. Find the length of diagonal \( \overline{AB} \) shown by filling in the steps. The first one is done for you.

**Step 1:** \( 3^2 + 6^2 = 9 + 36 = 45 \)

**Step 2:** diagonal of base = \( \sqrt{45} \approx \) _____________

**Step 3:** \( 4^2 + (\sqrt{45})^2 = (AB)^2 \)

**Step 4:** \( (AB)^2 = \) ________________

**Step 5:** \( AB \approx \) _____________ meters
The Pythagorean Theorem

In a right triangle,

the sum of the areas of the squares on the legs
is equal to
the area of the square on the hypotenuse.

\[ 3^2 + 4^2 = 5^2 \]
\[ 9 + 16 = 25 \]

Given the squares that are on the legs of a right triangle, draw the square for the hypotenuse below or on another sheet of paper.

1. leg   leg   hypotenuse

Without drawing the squares, you can find a missing leg or the hypotenuse when given the other sides.

Model  Example 1  Example 2

\[ a^2 + b^2 = c^2 \]
\[ 3^2 + 4^2 = c^2 \]
\[ 9 + 16 = c^2 \]
\[ 25 = c^2, \text{ so } c = 5 \text{ in.} \]

\[ a^2 + b^2 = c^2 \]
\[ a^2 + 12^2 = 15^2 \]
\[ a^2 = 225 - 144 \]
\[ a^2 = 81, \text{ so } a = 9 \text{ in.} \]

Find the missing side.

2.  8 in.  15 in.

3.  \[ \overline{BC} \]
\[ a \]
\[ 26 \text{ cm} \]
\[ \overline{AC} \]
\[ 24 \text{ cm} \]
The Pythagorean Theorem

When studying the **Pythagorean Theorem**, you have to use terms that are specific to right triangles.

There are two types of sides for a right triangle:

- **Legs**: The legs are perpendicular or at right angles to each other. The two legs may or may not be equal.

- **Hypotenuse**: The hypotenuse connects the ends of the two legs. Neither leg is as long as the hypotenuse.

In real-world problems, the **hypotenuse** is often called a **diagonal**.

- The diagonal often connects the opposite ends of two perpendicular line segments.
- In three-dimensional problems in which a **diagonal** is calculated, the Pythagorean Theorem often has to be used twice.

**Example**

To find the length of diagonal $AB$, two calculations have to be made.

- First, calculate the length of the diagonal connecting the two perpendicular sides that are 3 meters and 6 meters in length.
- Then use the first diagonal as a leg and the 4-meter side as the second leg to find the length of diagonal $AB$.

**Identify the parts of the triangle by describing or listing the sides.**

1. Hypotenuse: _______________________
   Legs: _______________________

2. Hypotenuse: _______________________
   Legs: _______________________

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The Pythagorean Theorem

Substitute values for \(a\), \(b\), or \(c\) in the Pythagorean Theorem to find the lengths of other sides.

\[
a^2 + b^2 = c^2
\]

If \(a = 2\) yards and \(b = 3\) yards
\[
2^2 + 3^2 = 4 + 9 = 13 = c^2
\]

To find \(c\), take the square root \((\sqrt{\cdot})\) of \(c^2\).
\[
c = \sqrt{13}
\]

Round your answer to the nearest tenth.
\[
c = \sqrt{13} \approx 3.6
\]

Problem 1

1. Name the parts of the right triangle:

   \[
   \begin{aligned}
   \text{Leg: } & \quad \text{___________________________} \\
   \text{Leg: } & \quad \text{___________________________} \\
   \text{Hypotenuse: } & \quad \text{___________________________}
   \end{aligned}
   \]

2. Fill in the blanks to find the length of the missing leg.

   \[
   \begin{align*}
   \text{Step 1} & \quad a^2 + b^2 = c^2 \\
   \text{Step 2} & \quad (\underline{\quad \quad \quad \quad})^2 + b^2 = (\underline{\quad \quad \quad \quad})^2 \\
   \text{Step 3} & \quad \underline{\quad \quad \quad \quad} + b^2 = \underline{\quad \quad \quad \quad} \\
   \text{Step 4} & \quad b^2 = \underline{\quad \quad \quad \quad} - \underline{\quad \quad \quad \quad} = \underline{\quad \quad \quad \quad} \\
   \text{Step 5} & \quad b = \sqrt{\underline{\quad \quad \quad \quad}} = \underline{\quad \quad \quad \quad}
   \end{align*}
   \]
Converse of the Pythagorean Theorem

Practice and Problem Solving: A/B

Write “yes” for sides that form right triangles and “no” for sides that do not form right triangles. Prove that each answer is correct.

1. 7, 24, 25
2. 30, 40, 45
3. 21.6, 28.8, 36

4. 10, 15, 18
5. 10.5, 36, 50
6. 2.5, 6, 6.5

Solve.

7. A commuter airline files a new route between two cities that are 400 kilometers apart. One of the two cities is 200 kilometers from a third city. The other one of the two cities is 300 kilometers from the third city. Do the paths between the three cities form a right triangle? Prove that your answer is correct.

8. A school wants to build a rectangular playground that will have a diagonal length of 75 yards. How wide can the playground be if the length has to be 30 yards?

9. A 250-foot length of fence is placed around a three-sided animal pen. Two of the sides of the pen are 100 feet long each. Does the fence form a right triangle? Prove that your answer is correct.
Problems 1–3 give the dimensions of isosceles triangles. Which triangles are right triangles? Prove your answer is correct.

1. Sides: 10
   Hypotenuse: 15

2. Sides: 2
   Hypotenuse: \(2\sqrt{2}\)

3. Sides: 300
   Hypotenuse: 700

Solve.

4. Can an equilateral triangle be a right triangle? Explain your answer.

5. A quadrilateral has two pair of opposite parallel sides. Each of one pair of parallel sides is 4 yards long. Each of the second pair of sides is 8 yards long. A diagonal connecting the opposite corners of the quadrilateral is 9 yards long. Is the quadrilateral a rectangle? Prove that your answer is correct.

6. A clear plastic prism has six faces, each of which is a parallelogram of side length 1 meter. A diagonal made of nylon filament line connecting two opposite vertices of the solid has a length of 4 meters. Is the solid a cube? Prove that your answer is correct.

A prism has a diagonal, \(d\), connecting its opposite vertices. Give the lengths of the sides of the prism in each problem that will make it the solid described.

7. \(d = 6\) meters; a cube

8. \(d = \sqrt{6}\) feet; a rectangular solid with a pair of opposite square faces and two pair of rectangular faces in which the length is twice the width.
Converse of the Pythagorean Theorem

Practice and Problem Solving: D

Problems 1–3 give the sides of a right triangle. In each case, which of the three sides is the hypotenuse? The first one is done for you.

1. 3, 4, 5
2. 5, 12, 13
3. 1, 1, \( \sqrt{2} \)
4. 2, 3, \( \sqrt{13} \)

Do the sides given in 5–8 form a right triangle? Prove your answer is correct using the Pythagorean Theorem, \( a^2 + b^2 = c^2 \).
The first one is done for you.

5. 8, 9, 10
6. 12, 14, 15

no; \( 8^2 + 9^2 \neq 10^2 \)

7. 10, 24, 26
8. 14, 15, 21

A parking lot has four sides. One pair of opposite sides are 100 yards long. The other two sides are 60 yards long. The distance from one end of the longer side to the opposite end of the shorter side is 120 yards. Is the parking lot a rectangle? Answer the questions below to find out.

9. If the distances given form a right triangle, which number is the hypotenuse, and why?

_________________________________________________________________________________________

10. Which numbers are the two sides?

_________________________________________________________________________________________

11. Fill in the numbers in the Pythagorean Theorem for this problem.

Does \( a^2 + b^2 = c^2 \)?

\( (\phantom{0})^2 + (\phantom{0})^2 \neq (\phantom{0})^2 \)

12. Simplify the numbers from problem 11.

Does \( \phantom{0} + \phantom{0} = \phantom{0}? \)

Does \( \phantom{0} = \phantom{0}? \)

13. Are the two sides of the equation equal? _______________


_________________________________________________________________________________________
**Converse of the Pythagorean Theorem**

**Reteach**

**Step 1** The first step in verifying that a triangle is a right triangle is to name the three sides. One side is the hypotenuse and the other two sides are legs.

- In a right triangle, the hypotenuse is opposite the right angle.
  - The hypotenuse is 5 cm.

- The hypotenuse is greater than either leg.
  - 5 cm > 4 cm and 5 cm > 3 cm

**Step 2** Next, the lengths of the hypotenuse and legs must satisfy the Pythagorean Theorem.

\[(\text{hypotenuse})^2 = (\text{first leg})^2 + (\text{second leg})^2\]

In the example above, \(5^2 = 3^2 + 4^2 = 25\), so the triangle is a right triangle.

**Conclusion** If the lengths of the hypotenuse and the two legs satisfy the conditions of the Pythagorean Theorem, then the triangle is a right triangle. If they do not satisfy the conditions of the Pythagorean Theorem, the triangle is not a right triangle.

Find the length of each hypotenuse.

1. 2. 

First, fill in the length of the hypotenuse in each problem. Then, determine if the sides form a right triangle.

3. 1, 2, 3 4. 8, 7, 6 5. 15, 20, 25

Hypotenuse: ________ Hypotenuse: ________ Hypotenuse: ________

Show that these sides form a right triangle.

6. 2, 3, \(\sqrt{13}\) 7. 3, 6, \(3\sqrt{5}\)
Converse of the Pythagorean Theorem

Reading Strategies: Draw Conclusions

Earlier, you learned about the Pythagorean Theorem. One way to state the Pythagorean Theorem is:

“If a triangle is a right triangle with legs \(a\) and \(b\) and a hypotenuse \(c\), then \(a^2 + b^2 = c^2\).”

In this lesson, you studied the converse of the Pythagorean Theorem. One way to state it is like this.

“If a triangle has sides of lengths \(a\), \(b\), and \(c\) and the lengths of the sides are related by the formula \(a^2 + b^2 = c^2\), then the triangle is a right triangle with a hypotenuse \(c\) and legs \(a\) and \(b\).”

In both cases, a condition is given that allows you to draw a conclusion. These are called “if-then” statements. Look at the examples below.

Example 1

A right triangle has a hypotenuse of 15 inches. One of its legs is 12 inches. How long is the other leg?

Solution

Using the Pythagorean Theorem, if a triangle is a right triangle, then its side lengths are related by the equation \(a^2 + b^2 = c^2\).

Substitute what you know: \(12^2 + b^2 = 15^2\)

Solve for what you want to find: \(b^2 = 15^2 - 12^2 = 225 - 144 = 81\)

Find \(b\): \(b = \sqrt{81} = 9\); the second leg is 9 inches.

Example 2

A triangle has sides of 3 meters, 5 meters, and 7 meters. Is it a right triangle?

Solution

Using the converse of the Pythagorean Theorem, if the sides of a triangle satisfy the formula \(a^2 + b^2 = c^2\), then the triangle is a right triangle.

Substitute: \(3^2 + 5^2 = 34; 7^2 = 49\)

Does \(a^2 + b^2 = c^2\)? No. You can draw the conclusion that a triangle with sides of lengths 3 meters, 5 meters, and 7 meters is not a right triangle.

Write an “if-then” statement for each problem using the Pythagorean Theorem and its converse as shown above. Then use your statement to answer the question.

1. A right triangle has sides of 5 and 12. What is its third side?
   “If _______________________,
   then _______________________.”

2. A triangle has sides of lengths 4, 4, and 6. Is it a right triangle?
   “If _______________________,
   then _______________________.”
Problem 1

1. The **converse** of the Pythagorean Theorem is the “opposite” of the Pythagorean Theorem. It states that if the square of the hypotenuse equals the sum of the squares of the other two sides of a triangle, then the triangle is a right triangle. Explain how Problem 1 is an example of the converse of the Pythagorean Theorem.

_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________

Problem 2

2. In Problem 2, does the hypotenuse need to be longer or shorter to form a right triangle?
LESSON 12-3  Distance Between Two Points

Practice and Problem Solving: A/B

Name the coordinates of the points.
1.  
   \[
   A(\text{_____}, \text{_____}) \quad \text{B(_____}, \text{_____}) \quad \text{C(_____}, \text{_____})
   \]

2.  
   \[
   D(\text{_____}, \text{_____}) \quad \text{E(_____}, \text{_____}) \quad \text{F(_____}, \text{_____})
   \]

Name the hypotenuse of each right triangle in problems 1 and 2.
3. Hypotenuse in problem 1: 
4. Hypotenuse in problem 2: 

Estimate the length of the hypotenuse for each right triangle in problems 1 and 2.
5. Hypotenuse in problem 1: 
6. Hypotenuse in problem 2: 

Use the distance formula to calculate the length of the hypotenuse for each right triangle.
7. Hypotenuse in problem 1: 
8. Hypotenuse in problem 2: 

9. Use the distance formula to find the distance between the points
   \((-4, -4) \text{ and } (4, 4)\).
LESSON 12-3 Distance Between Two Points

Practice and Problem Solving: C

1. Use the distance formula to determine if the three points, \(A(1, 3)\), \(B(-2, 4)\), and \(C(1, 4)\), form a right triangle.

_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________

2. A triangular-shaped forest preserve is formed by roads \(AB\), \(BC\), and \(CA\) as shown on the map. Find its perimeter using the distance formula. The distances are in kilometers.

_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________

The distance \(d\) between two points is 5. Find the missing coordinate of the second point that will give this distance. Show your work.

3. \(A(-5, 7); B(x, 3)\)

_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________

4. \(C(3, -4); D(6, y)\)

_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________
Distance Between Two Points

Practice and Problem Solving: D

Find the distance between the points by filling in the steps. The first one is done for you.

1. \(A(1, 2)\) and \(B(3, 4)\)
   \[
   d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}; \quad x_2 = 3; \quad x_1 = 1; \quad y_2 = 4; \quad y_1 = 2
   \]
   \[
   = \sqrt{(3 - 1)^2 + (4 - 2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}
   \]

2. \(C(-1, 3)\) and \(D(-5, 7)\)
   \[
   d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}; \quad x_2 = \_\_\_; \quad x_1 = \_\_\_; \quad y_2 = \_\_\_; \quad y_1 = \_\_\_
   \]

3. \(E(0, -5)\) and \(F(10, -15)\)
   \[
   d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}; \quad x_2 = \_\_\_; \quad x_1 = \_\_\_; \quad y_2 = \_\_\_; \quad y_1 = \_\_\_
   \]

Use the graph to estimate the distance between the points. The first one is done for you.

4. \(A\) and \(B\)
   \[
   x\text{-distance between points} = \_\_\_; \quad AB = \_\_\_; \quad AB = \text{more than 10}
   \]

5. \(C\) and \(D\)

6. \(E\) and \(F\)

7. \(F\) and \(G\)

If the \(x\)-coordinates of two points are equal, the distance between the points can be found by taking the absolute value of the difference of the \(y\)-coordinates of the two points. Find the distance between the points. The first one is done for you.

8. \(A(-2, 5)\) and \(B(-2, 1)\)
   \[
   \text{The difference of the } y\text{-coordinates is } |5 - 1| \quad \text{or } 4.
   \]

9. \(C(1, -4)\) and \(D(1, -1)\)

10. \(E(0, -6)\) and \(F(0, 9)\)
Distance Between Two Points

There are three cases of distance between two points on a coordinate plane. The first two have fewer steps than the third, but you have to be able to identify when to use them.

**Case 1**
The x-coordinates of the two points are the same.
If the x-coordinates are the same, the distance between the two points is the absolute value of the difference of the y-coordinates.
The line connecting the two points is a vertical line.

**Example 1**
Find the distance between the two points $A(-3, 5)$ and $B(-3, -4)$.
$$\text{The x-coordinates are the same.}$$
$$\text{Difference of the y-coordinates: } 5 - (-4) = 9$$
$$\text{The absolute value of 9 is 9.}$$

**Case 2**
The y-coordinates of the two points are the same.
If the y-coordinates are the same, the distance between the two points is the absolute value of the difference of the x-coordinates.
The line connecting the two points is a horizontal line.

**Example 2**
Find the distance between the two points $C(1, 3)$ and $D(6, 3)$.
$$\text{The y-coordinates are the same.}$$
$$\text{Difference of the x-coordinates: } 1 - 6 = -5$$
$$\text{The absolute value of -5 is 5.}$$

**Case 3**
If the x and y-coordinates of the two points are different, use the distance formula:
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
The x and y-coordinates are different if $x_1 \neq x_2$ and $y_1 \neq y_2$.
The line connecting the two points can be thought of as the hypotenuse of a right triangle.

**Example 3**
Find the distance between the two points $E(-9, 5)$ and $F(-4, 0)$.
$$\text{Use the distance formula.}$$
$$d = \sqrt{(-4 + 9)^2 + (0 - 5)^2}$$
$$= \sqrt{5^2 + (-5)^2} = 5\sqrt{2}$$

Tell whether the points given are endpoints of a vertical line, a horizontal line, or neither.

1. $(-8, 1)$, $(-5, 1)$
2. $(4, 3)$, $(2, 1)$
3. $(0, 0)$, $(0, 100)$
4. $(3, 3)$, $(3, 3)$

Use the distance formula to find the distance between the two points.

5. $(0.5, 1.3)$, $(-0.4, -1.2)$
6. $(6, -3)$, $(2, -4)$
Distance Between Two Points

Reading Strategies: Identify Relationships

Look at the triangles shown below.

1. Are the triangles the same size? __________________________

2. What formula should you use to find the length of the hypotenuse for the triangle on the left? __________________________

3. What formula should you use to find the length of the hypotenuse for the triangle on the right? __________________________

4. Using the Pythagorean Theorem:
   \[ a^2 + b^2 = c^2 \]

5. Using the Distance Formula:
   \[ d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \]

6. What do you notice about the last two steps of the formulas above?
   __________________________________________________________________________________________
   __________________________________________________________________________________________
Distance Between Two Points

**Problem 1**

How do you know which coordinate is which in the distance formula?

Here are two points: \(A(1, 2)\) and \(B(4, 7)\).

**Step 1** The Distance Formula: 
\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

**Step 2** What is \(x_1\)? 
\(x_1\) is the \(x\)-coordinate of point \(A\), the first point.

**Step 3** What is \(x_2\)? 
\(x_2\) is the \(x\)-coordinate of point \(B\).

**Problem 2**

Find the distance between the two points using the Distance Formula.

Here’s the formula: 
\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Substitute the numbers from Problem 1: 
\[ d = \sqrt{(4 - 1)^2 + (7 - 2)^2} \]

Simplify: 
\[ d = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34} \]

You can leave the answer as a square root, or you can use a calculator to find that the square root of 34 is about 5.8.

**Name** \(x_1\), \(x_2\), \(y_1\), and \(y_2\). Then, find the distance between the points.

1. \(C(6, 4)\) and \(D(9, 5)\)
2. \(X(0, 6)\) and \(Y(1, 8)\)

\[ x_1: \quad ; \quad x_2: \quad ; \quad y_1: \quad ; \quad y_2: \quad \]

\[ x_1: \quad ; \quad x_2: \quad ; \quad y_1: \quad ; \quad y_2: \quad \]

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
The Pythagorean Theorem

Pythagoras Without Squares

The Pythagorean Theorem describes squares constructed on the sides of a right triangle. But, the constructed figures do not necessarily need to be square. See what happens for similar figures of different shapes.

1. Show that the Pythagorean Theorem is true for a 3-4-5 right triangle. Then write the general formula using $a$, $b$, and $c$ for the side lengths, where $c$ is the hypotenuse.

2. In Figure 1, the three isosceles right triangles are each one-half of a square. Show that the area sum $A + B$ equals the area of $C$ for this triangle. The general formula is shown below Figure 1.

For each figure, the area sum $A + B$ equals the area $C$ on the hypotenuse. Show this is true for the specific figure. Then write a general formula using $a$, $b$, and $c$ for the side lengths.

3. Figure 2

4. Figure 3

5. Create your own variation on the Pythagorean Theorem by constructing similar figures on the sides of a right triangle.
Volume of Cylinders

Practice and Problem Solving: A/B

Find the volume of each cylinder. Round your answer to the nearest tenth if necessary. Use 3.14 for $\pi$.

1. 6.5 cm
   [Diagram of a cylinder with a height of 16 cm]

2. 4 in.
   [Diagram of a cylinder with a height of 3 in.]

3. A cylindrical oil drum has a diameter of 2 feet and a height of 3 feet. What is the volume of the oil drum?

4. New Oats cereal is packaged in a cardboard cylinder. The packaging is 10 inches tall with a diameter of 3 inches. What is the volume of the New Oats cereal package?

5. A small plastic storage container is in the shape of a cylinder. It has a diameter of 7.6 centimeters and a height of 3 centimeters. What is the volume of the storage cylinder?

6. A can of juice has a diameter of 6.6 centimeters and a height of 12.1 centimeters. What is the total volume of a six-pack of juice cans?

7. Mr. Macady has an old cylindrical grain silo on his farm that stands 25 feet high with a diameter of 10 feet. Mr. Macady is planning to tear down the old silo and replace it with a new and bigger one. The new cylindrical silo will stand 30 feet high and have a diameter of 15 feet.
   a. What is the volume of the old silo?
   b. What is the volume of the new silo?
   c. How much greater is the volume of the new silo than the old silo?
**Volume of Cylinders**

**Practice and Problem Solving: C**

Solve. Use 3.14 for \( \pi \). Round your answer to the nearest tenth, if necessary. Show your work.

1. A feeding trough was made by hollowing out half of a log. The trough is shaped like half a cylinder. It is 5 feet long and has an interior diameter of 1.5 feet. What is the volume of oats that will fill the trough?

_________________________________________________________________________________________

2. You buy 3 candles in the shape of cylinders as shown below.

![Candle Diagrams](image)

   a. Use the dimensions shown above to find the volume of each candle to the nearest hundredth. Complete the table.

<table>
<thead>
<tr>
<th>Candle Size</th>
<th>Radius</th>
<th>Height</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tall Candle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medium Candle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short Candle</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Which candle has the most wax?

_____________________________________________________________________________________

   c. How much more wax does this candle have than the candle with the least wax?

_____________________________________________________________________________________

   d. Write an inequality that shows the amount of wax in the three candles ordered from least to greatest.

_____________________________________________________________________________________

3. A cylinder has a height of 8 feet and a volume of 628 cubic feet. Find the radius of the cylinder. Use 3.14 for \( \pi \). Show your work.

_________________________________________________________________________________________

_________________________________________________________________________________________

_________________________________________________________________________________________
**LESSON 13-1 Volume of Cylinders**

*Practice and Problem Solving: D*

Find the volume of each cylinder. Round your answer to the nearest tenth if necessary. Use 3.14 for $\pi$. Show your work by filling in the blanks with values from the diagrams. The first one is done for you.

1. \[ V = \pi r^2 h \]
   \[ V = 3.14 \cdot 4^2 \times 9 \]
   \[ V = 3.14 \cdot 16 \times 9 \]
   \[ V \approx 452.2 \text{ in}^3 \]

2. \[ V = \pi r^2 h \]
   \[ V = \_ \cdot \_ \times \_ \]
   \[ V \approx \_ \]

Find the volume of each cylinder. Round your answer to the nearest tenth if necessary. Use 3.14 for $\pi$. The first one is done for you.

3. \[ 4 \text{ cm} \]
   \[ 3 \text{ cm} \]
   \[ 150.7 \text{ cm}^3 \]

4. \[ 5 \text{ cm} \]
   \[ 10 \text{ cm} \]

**Solve. The first one is done for you.**

5. A can of beans is 4.5 inches high and has a diameter of 3 inches. Find the volume of the can to the nearest tenth of a unit. Use 3.14 for $\pi$.
   \[ 31.8 \text{ in}^3 \]

6. A telephone pole is 30 feet tall with a diameter of 12 inches. Jacob is making a replica of a telephone pole and wants to fill it with sand to help it stand freely. Find the volume of his model, which has a height of 30 inches and a diameter of 1 inch, to the nearest tenth of a unit. Use 3.14 for $\pi$. 

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Volume of Cylinders

Reteach

You can use your knowledge of how to find the area of a circle to find the volume of a cylinder.

1. What is the shape of the base of the cylinder?  
   circle

2. The area of the base is \( B = \pi r^2 \).
   \[ B = 3.14 \cdot 1^2 = 3.14 \text{ cm}^2 \]

3. The height of the cylinder is __ cm.

4. The volume of the cylinder is \( V = B \cdot h \).
   \[ V = 3.14 \cdot 5 = 15.7 \text{ cm}^3 \]

The volume of the cylinder is 15.7 cm³.

---

1. a. What is the area of the base?  
   \[ B = 3.14 \cdot ____^2 = ____ \text{ cm}^2 \]

   b. What is the height of the cylinder? ____ cm

   c. What is the volume of the cylinder?  
   \[ V = B \cdot h = ____ \cdot ____ = ____ \text{ cm}^3 \]

2. a. What is the area of the base?  
   \[ B = 3.14 \cdot ____^2 = ____ \text{ cm}^2 \]

   b. What is the height of the cylinder? ____ cm

   c. What is the volume of the cylinder?  
   \[ V = B \cdot h = ____ \cdot ____ = ____ \text{ cm}^3 \]
Volume of Cylinders

Reading Strategies: Use Diagrams

You can use diagrams to help solve problems. Use the information from the diagram below to find the area of the base and volume of a cylinder.

Example 1

What is the area of the base of the cylinder?

Step 1: Write the formula for the area of the base of a cylinder.

$$B = \pi r^2$$

Step 2: Substitute the values you know. Evaluate.

$$B = 3.14 \times (5)^2$$
$$B = 3.14 \times 25$$
$$B = 78.5 \text{ cm}^2$$

Example 2

What is the volume of the cylinder?

Step 1: Write the formula for the volume of a cylinder.

$$V = Bh \text{ or } V = \pi r^2 h$$

Step 2: Substitute the values you know. Evaluate.

$$V = \frac{78.5}{7} \times 2$$
$$V = 157 \text{ cm}^3$$

Solve.

1. What is the area of the base of the cylinder?

2. What is the volume of the cylinder?
Volume of Cylinders
Success for English Learners

Problem 1
Find the area of the base of the cylinder.

Step 1
The base of a cylinder is a circle, so use the formula for the area of a circle.
\[ B = \pi r^2 \]

Step 2
Substitute the values. Use 3.14 for \( \pi \). Then evaluate.
\[ B \approx 3.14 \times 4^2 \]
\[ B \approx 3.14 \times 16 \]
\[ B \approx 50.24 \text{ in}^2 \]

Problem 2
Find the volume of the cylinder. Round to the nearest tenth, if necessary.

Step 1
Multiply the area of the base \( (B) \) by the height \( (h) \) of the cylinder to find its volume. Use the formula for the volume of a cylinder.
\[ V = Bh \]

Step 2
Substitute the values. Then evaluate.
\[ V = (50.24 \text{ in}^2) \times (7 \text{ in.}) \]
\[ V = 351.68 \text{ in}^3 \]

1. What formula is the same as \( B = \pi r^2 \)?

_________________________________________________________________________________________
_________________________________________________________________________________________

2. Why is the volume of the cylinder expressed in cubic inches?

_________________________________________________________________________________________
_________________________________________________________________________________________
_________________________________________________________________________________________

3. Write a problem of your own about finding the volume of a cylinder. Make a sketch of your cylinder at the right. Then solve the problem.
Volume of Cones

Practice and Problem Solving: A/B

Find the volume of each cone. Round your answer to the nearest tenth if necessary. Use 3.14 for π.

1. \[ \text{Volume of cone: } \]

2. \[ \text{Volume of cone: } \]

3. The mold for a cone has a diameter of 4 inches and is 6 inches tall. What is the volume of the cone mold to the nearest tenth?

4. A medium-sized paper cone has a diameter of 8 centimeters and a height of 10 centimeters. What is the volume of the cone?

5. A funnel has a diameter of 9 in. and is 16 in. tall. A plug is put at the open end of the funnel. What is the volume of the cone to the nearest tenth?

6. A party hat has a diameter of 10 cm and is 15 cm tall. What is the volume of the hat?

7. Find the volume of the composite figure to the nearest tenth. Use 3.14 for π.
   a. Volume of cone:
   b. Volume of cylinder:
   c. Volume of composite figure:
Volume of Cones

Practice and Problem Solving: C

Find the volume of each cone. Use 3.14 for \( \pi \). Round your answer to the nearest tenth, if necessary. Show your work.

1. Lucas makes models of cones to explore how changing dimensions affect volume. Cone A is 10 centimeters high and its base has a diameter of 4 centimeters. Cone B is twice as tall with a height of 20 centimeters and a diameter of 4 centimeters. Cone C is the same height as Cone A, 10 centimeters, but the diameter of its base is 8 centimeters. Complete the table below.
   a. Use the dimensions shown above to find the volume of each cone to the nearest hundredth. Complete the table.

<table>
<thead>
<tr>
<th>Cone Size</th>
<th>Radius</th>
<th>Height</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cone A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cone B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cone C</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Which cone has the greatest volume?

   c. How much greater is the volume of Cone B than that of Cone A?

   d. Write an inequality that shows the volume of all three cones ordered from greatest to least.

   e. Does the volume of a cone increase more when you double the height of the original or when you double its radius? Explain why.

Solve.

2. A large traffic cone stands 28 inches in height and has a volume of 732.7 cubic inches. What is the diameter of the base of the cone? Use 3.14 for \( \pi \). Show your work.
**Volume of Cones**

**Practice and Problem Solving: D**

Find the volume of each cone. Round your answer to the nearest tenth if necessary. Use 3.14 for $\pi$. Show your work by filling in the blanks with values from the diagrams. The first one is done for you.

1. $V = \frac{1}{3} \pi r^2 h$
   
   $V = \frac{1}{3} \cdot 3.14 \cdot 3^2 \times 5$
   
   $V = \frac{1}{3} \cdot 3.14 \cdot 9 \times 5$
   
   $V = 47.1 \text{ cm}^3$

2. $V = \frac{1}{3} \pi r^2 h$
   
   $V = \frac{1}{3} \cdot \text{_____} \cdot \text{_____} \times \text{_____}$
   
   $V = \text{_____}$

Find the volume of each cone. Round your answer to the nearest tenth if necessary. Use 3.14 for $\pi$. The first one is done for you.

3. $V = \frac{1}{3} \pi r^2 h$
   
   $V = \frac{1}{3} \cdot 3.14 \cdot 4 \times 6$
   
   $V = 100.5 \text{ in}^3$

4. $V = \frac{1}{3} \pi r^2 h$
   
   $V = \frac{1}{3} \cdot \text{_____} \cdot \text{_____} \times \text{_____}$
   
   $V = \text{_____}$

**Solve. The first one is done for you.**

5. A cone has a diameter of 4 cm and a height of 11 cm. What is the volume of the cone to the nearest tenth? Use 3.14 for $\pi$.
   
   $46.1 \text{ cm}^3$

6. A cloth pastry bag is shaped like a cone. It has a radius of 1.5 inches and a height of 8.5 inches. What is the volume of the pastry bag to the nearest tenth? Use 3.14 for $\pi$.
   
   $\text{_____}$
You can use your knowledge of how to find the volume of a cylinder to help find the volume of a cone.

This cone and cylinder have congruent bases and congruent heights.

Volume of Cone $= \frac{1}{3}$ Volume of Cylinder

Use this formula to find the volume of a cone.

$$V = \frac{1}{3} Bh$$

Complete to find the volume of each cone.

1. $h = 10\text{ in.}$

radius $r$ of base = ____ in.

$$V = \frac{1}{3} Bh$$

$$V = \frac{1}{3} (\pi r^2)h$$

$$V = \frac{1}{3} (\pi \times \underline{\hspace{1cm}}) \times \underline{\hspace{1cm}}$$

$$V = \frac{1}{3} (\underline{\hspace{1cm}}) \times \underline{\hspace{1cm}}$$

$$V = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$$

$$V = \underline{\hspace{1cm}}$$

$$V \approx \underline{\hspace{1cm}} \text{ in}^3$$

2. $d = 12\text{ cm}$

radius $r = \frac{1}{2}$ diameter = ____ cm

$$V = \frac{1}{3} Bh$$

$$V = \frac{1}{3} (\pi r^2)h$$

$$V = \frac{1}{3} (\pi \times \underline{\hspace{1cm}}) \times \underline{\hspace{1cm}}$$

$$V = \frac{1}{3} (\underline{\hspace{1cm}}) \times \underline{\hspace{1cm}}$$

$$V = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$$

$$V = \underline{\hspace{1cm}}$$

$$V \approx \underline{\hspace{1cm}} \text{ cm}^3$$
Volume of Cones

Reading Strategies: Use Diagrams

Reading a diagram accurately is important when solving problems.

THINK!
- The base is shaded to show its area.
- The diameter and the radius of the base are shown by a solid line.
- The height of the cone is shown by a dashed line.
- Find the area of the base by using the formula $B = \pi r^2$.
- Check the diagram to see if it shows the diameter or the radius of the base of the cone.
- The formula asks for the radius. Remember that the diameter is twice the radius. So, $d = 2r$ and $r = \frac{d}{2}$

Use the diagrams to answer the questions. Use 3.14 for $\pi$. Round your answer to the nearest tenth, if necessary.

1. Does the diagram of Cone A show the radius or the diameter of the base?

2. What is the radius of the base of Cone A?

3. What is the area of the base of Cone A?

4. What is the height of Cone A?

5. What is the volume of Cone A?

6. What is the volume of Cone B?
**Problem 1**

What is the area of the base of this cone?

![Diagram of a cone with a radius of 4 inches and a height of 6 inches.](image)

Think about the values the diagram gives you and the values you need. Are they the same?

\[
B = \pi r^2
\]

\[
B = 3.14 \cdot 16
\]

\[
B = 50.24 \text{ in}^2
\]

**Problem 2**

What is the volume of this cone?

![Diagram of a cone with a radius of 4 inches and a height of 6 inches.](image)

Think about the values you must substitute.

\[
V = \frac{1}{3} Bh
\]

\[
V = \frac{1}{3} \cdot 50.24 \cdot 6
\]

\[
V = 100.48 \text{ in}^3
\]

1. What must you do to find the radius of the base of a cone when a diagram gives you the measure of its diameter?

2. Write a problem of your own about finding the volume of a paperweight in the shape of a cone. Then, solve.
Volume of Spheres

Practice and Problem Solving: A/B

Find the volume of each sphere. Round your answer to the nearest tenth if necessary. \( V = \frac{4}{3} \pi r^3 \). Use 3.14 for \( \pi \). Show your work.

1. 

2. 

3. \( r = 3 \) inches

4. \( d = 9 \) feet

5. \( r = 1.5 \) meters

6. A globe is a map of Earth shaped as a sphere. What is the volume, to the nearest tenth, of a globe with a diameter of 16 inches?

7. The maximum diameter of a bowling ball is 8.6 inches. What is the volume to the nearest tenth of a bowling ball with this diameter?

8. According to the National Collegiate Athletic Association men's rules, a tennis ball must have a diameter of more than \( 2 \frac{1}{2} \) inches and less than \( 2 \frac{5}{8} \) inches.

   a. What is the volume of a sphere with a diameter of \( 2 \frac{1}{2} \) inches?

   b. What is the volume of a sphere with a diameter of \( 2 \frac{5}{8} \) inches?

   c. Write an inequality that expresses the range in the volume of acceptable tennis balls.
Volume of Spheres

Find the volume of each sphere. Use 3.14 for $\pi$. $V = \frac{4}{3} \pi r^3$. Show your work.

1. A basic sphere has a diameter of 6 inches, a mini sphere has a diameter of 3 inches, and a maxi sphere has a diameter of 12 inches. Complete the table below.

   a. Use the dimensions shown above to find the volume of each sphere to the nearest hundredth. Complete the table.

<table>
<thead>
<tr>
<th>Sphere Size</th>
<th>Radius</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Sphere</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mini Sphere</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maxi Sphere</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. How does the volume of the mini sphere compare to the volume of the basic sphere?

   c. How does the volume of the maxi sphere compare to the volume of the basic sphere?

   d. Write an inequality that shows the volume of all 3 spheres ordered from least to greatest.

2. What is the radius of a sphere with a volume of 4186 in$^3$ to the nearest tenth of an inch?

3. If the radius of a sphere is equal to the length of the sides of a cube, are their volumes equal? Why or why not?
Lesson 13-3

Volume of Spheres

Practice and Problem Solving: D

Find the volume of each sphere. Round your answer to the nearest tenth if necessary. Use 3.14 for π. Show your work by filling in the blanks with values from the diagrams. The first one is done for you.

1.  
   \[ V = \frac{4}{3} \pi r^3 \]
   \[ V = \frac{4}{3} \times 3.14 \times 9^3 \]
   \[ V \approx 3,052.1 \text{ cm}^3 \]

2.  
   \[ V = \frac{4}{3} \pi r^3 \]
   \[ V = \frac{4}{3} \times 3.14 \times \text{2 m}^3 \]
   \[ V = \text{ } \]

Find the volume of each sphere. Round your answer to the nearest tenth if necessary. Use 3.14 for π. The first one is done for you.

3.  
   \[ V = \frac{4}{3} \pi r^3 \]
   \[ V = \frac{4}{3} \times 3.14 \times 8^3 \]
   \[ V = 267.9 \text{ cm}^3 \]

4.  
   \[ V = \frac{4}{3} \pi r^3 \]
   \[ V = \frac{4}{3} \times 3.14 \times 10^3 \]
   \[ V = \text{ } \]

Solve. The first one is done for you.

5. What is the volume to the nearest tenth of a spherical scoop of frozen yogurt with a diameter of 5.6 cm? Use 3.14 for π.
   \[ V = \frac{4}{3} \pi \left(\frac{5.6}{2}\right)^3 \]
   \[ V = 91.9 \text{ cm}^3 \]

6. Mike makes homemade apple lollipops. Each lollipop has a diameter of 2 in. What is the volume of the lollipop to the nearest tenth? Use 3.14 for π.
   \[ V = \frac{4}{3} \pi \left(\frac{2}{2}\right)^3 \]
LESSON 13-3  Volume of Spheres

Reteach

• All points on a sphere are the same distance from its center.
• Any line drawn from the center of a sphere to its surface is a radius of the sphere.
• The radius is half the measure of the diameter.
• Use this formula to find the volume of a sphere.

\[ V = \frac{4}{3} \pi r^3 \]

Complete to find the volume of each sphere to the nearest tenth. Use 3.14 for \( \pi \). The first one is done for you.

1. A regular tennis ball has a diameter of 2.5 inches.
   
   \[
   \text{diameter} = 2.5 \text{ inches} \\
   \text{radius} = 1.25 \text{ inches}
   \]

   \[
   V = \frac{4}{3} \pi (1.25)^3 \\
   V = \frac{4}{3} \cdot 3.14 \cdot 1.95 \\
   V = 8.164 \\
   V \approx 8.2 \text{ in}^3
   \]

2. A large grapefruit has a diameter of 12 centimeters.
   
   \[
   \text{diameter} = 12 \text{ cm} \\
   \text{radius} = 6 \text{ cm}
   \]

   \[
   V = \frac{4}{3} \pi (6)^3 \\
   V = \frac{4}{3} \cdot 3.14 \cdot 216 \\
   V = 301.536 \\
   V \approx ______
   \]
Volume of Spheres

Reading Strategies: Build Vocabulary

Use the diagram and vocabulary to answer the questions.

1. What is one example of a real-world object shaped like a sphere?

_________________________________________________________________________________________

2. What passes through the center of a sphere and touches the surface at two points?

_________________________________________________________________________________________

3. What connects the center of a sphere to one point on the surface?

_________________________________________________________________________________________

4. If you are given the diameter of a sphere, how can you find its radius?

_________________________________________________________________________________________

5. If you are given the radius of a sphere, how can you find its diameter?

_________________________________________________________________________________________

Sphere: A 3-D figure shaped like a ball.

Diameter: Passes through the center and connects two points on the surface.

Radius: Connects the center with one point on the surface.

Think!

Diameter $d = \text{twice radius } r$
Volume of Spheres

Success for English Learners

Problem 1

What is the volume of a sphere with a diameter of 20 cm?

Use the formula $V = \frac{4}{3} \pi r^3$. Use 3.14 for $\pi$.

Round your answer to the nearest tenth.

The radius is half the diameter.

$20 \div 2 = 10$

$r = 10$ cm

What is $r^3$?

$10 \cdot 10 \cdot 10 = 1,000$

$r^3 = 1,000$ cm$^3$

What does the formula for the volume of a sphere look like when you plug in numbers from the problem?

$V = \frac{4}{3} \pi r^3$

$V = \frac{4 \cdot 3.14 \cdot 1,000}{3}$

What operations must you perform next to find the volume of the sphere?

First, multiply. $\Rightarrow$ Next, divide. $\Rightarrow$ Finally, round

$V = \frac{12,560}{3}$

$V = 4186.66...$

$V \approx 4186.7$ cm$^3$

1. What does cm$^3$ in the answer mean?

2. Write a problem of your own about finding the volume of a sphere. Then, solve.
Volume

Challenge

Use 3.14 for $\pi$.

1. Design a cylinder that has a volume of between 430 and 450 cubic inches. Sketch the cylinder and label the radius or diameter and the height. Prove your cylinder meets these conditions by showing your calculations.

2. Design a cone that has a volume of between 90 and 100 cubic inches. Sketch the cone and label the radius or diameter and the height. Prove your cone meets these conditions by showing your calculations.

3. Design a sphere that has a volume of between 980 and 1,005 cubic centimeters. Sketch the sphere and label the radius or diameter. Prove your sphere meets these conditions by showing your calculations.
UNIT 5: Measurement Geometry

MODULE 11 Angle Relationships in Parallel Lines and Triangles

LESSON 11-1

Practice and Problem Solving: A/B
1. $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$
2. $\angle 1$
3. alternate exterior angles
4. Accept: same-side interior angles OR supplementary angles.
5. Accept $\angle 2$ OR $\angle 6$.
6. Accept: $\angle 4$ OR $\angle 8$.
7. 146°
8. 128°
9. 135°
10. 45°
11. $\angle 2$ and $\angle 7$, $\angle 1$ and $\angle 8$
12. $\angle 8$
13. alternate interior angles
14. Yes; Accept any two pairs of supplementary angles.

Practice and Problem Solving: C
1. Avenue A and South Street are also perpendicular. Possible explanation: The measure of the angles formed by Avenue A and North Street is 90°. Then the measure of corresponding angles is also 90°, making Avenue A and South Street perpendicular.
2. 70° and 110°
3. $(180 - 5x)°$
4. $m\angle BCF = 48°$; $m\angle CFE = 132°$
5. $m\angle CFG = 42°$; $m\angle DCF = 138°$
6–8. Check students’ work.

Practice and Problem Solving: D
1. The lines are parallel.
2. $a$ and $b$
3. $c$

4. $\angle 1$ and $\angle 5$
5. $\angle 4$
6. $\angle 1$, $\angle 3$, or $\angle 5$
7. 40°
8. 30°
9. 35°
10. 130°
11. Check students’ drawings.
   Sample answer:

   ![Diagram](image)

Reteach
1. $\angle 3$, $\angle 5$, $\angle 7$
2. $\angle c$, $\angle e$, $\angle g$
3. $\angle y$, $\angle t$, $\angle r$
4. 
   ![Diagram](image)

5. 
   ![Diagram](image)

6. 
   ![Diagram](image)
Reading Strategies
1. Check students’ work. A pair of corresponding angles should be labeled 1 and 2.
   Sample answer:
   \[ \angle 1 \text{ and } \angle 2 \]
2. Check students’ work. A pair of alternate interior angles should be labeled 1 and 2.
   Sample answer:
   \[ \angle 3 \text{ and } \angle 6 \]
3. Check students’ work. A pair of alternate exterior angles should be labeled 1 and 2.
   Sample answer:
   \[ \angle 1 \text{ and } \angle 8 \]

Success for English Learners
1. \( \angle 1 \) and \( \angle 5 \), \( \angle 2 \) and \( \angle 6 \), \( \angle 3 \) and \( \angle 7 \), \( \angle 4 \) and \( \angle 8 \)
2. \( \angle 3 \) and \( \angle 6 \), \( \angle 4 \) and \( \angle 5 \)
3. \( \angle 1 \) and \( \angle 8 \), \( \angle 2 \) and \( \angle 7 \)

LESSON 11-2

Practice and Problem Solving: A/B
1. A
2. B
3. 30°
4. 46°
5. 55°
6. \( x = 40° \)
7. \( y = 65° \)
8. \( n = 65° \)
9. 103°
10. 77°
11. 60°

Practice and Problem Solving: C
1. \( x = 59° \)
2. \( n = 46° \)
3. \( t = 60° \)
4. \( w = 31° \)
5. \( x = 50° \)
6. \( x = 30° \)
7. \( 180 = (4x - 9) + (4x - 9) + x; \) base angles = 79°; vertex angle = 22°
8. \( 180 = 2x + \frac{x}{4}; \) base angles = 80°; vertex angle = 20°
9. \( 180 = x + 2x + 3x; 30°, 60°, 90° \)

Practice and Problem Solving: D
1. 55°
2. 136°
3. 74°
4. 16°
5. 40°
6. 112°
7. 103°
8. 68°
9. 82°
10. \( x = 65° \)
11. \( y = 40° \)
12. \( r = 30° \)

Reteach
1. \( 55s + 72° = 127°; 180° - 127° = 53°; 53° \)
2. \( 82° + 53° = 135°; 180° - 135° = 45°; 45° \)
3. \( y = 150° \)
4. 150°; 30°

Reading Strategies
1. 40°
2. 75°, 65°, 40°
3. 75°

Success for English Learners
1. \( x = 80° \)
2. \( x = 58° \)
LESSON 11-3

Practice and Problem Solving: A/B
1. \( \triangle ABC \) has angle measures 42°, 50°, 88°, and \( \triangle FGH \) has angle measures 42°, 50°, 88°. The triangles are similar because two angles in one triangle are congruent to two angles in the other triangle.
2. \( \triangle Xyz \) has angle measures 41°, 55°, 84°, and \( \triangle PRQ \) has angle measures 38°, 55°, 87°. The triangles are not similar because the triangles have only one congruent pair of angles.
3. Both triangles contain both \( \angle N \) and a right angle, so \( \triangle LQN \) is similar to \( \triangle MPN \).
4. 4 ft
5. No; \( \angle J \) is in both \( \triangle LQJ \) and \( \triangle KRJ \), but there is not enough information given to find any other congruent angles. \( \angle R \) looks like a right angle, but it is not given.
6. \( \angle TSV \) and \( \angle TRW \) are congruent because they are corresponding angles, and both triangles contain \( \angle T \). By AA similarity, \( \triangle RTW \) is similar to \( \triangle STV \).

Practice and Problem Solving: C
1. \( \triangle XYZ \) and \( \triangle RQP \) are similar. The triangles are similar because two angles in one triangle are congruent to two angles in the other triangle. Both triangles have angles measures 32°, 84°, 64°.
2. No, similar triangles have congruent corresponding angles. However, corresponding sides of similar triangles are proportional, not congruent.
3. 17.5 ft
4. 20 ft
5. \( \angle BCA \) and \( \angle GHF \) are congruent because they are corresponding angles, and both triangles contain right angles. By AA similarity, \( \triangle ABC \) is similar to \( \triangle FGH \).
6. \( H(18, 16) \)

Practice and Problem Solving: D
1. \( m \angle C = 59° \)
2. \( m \angle P = 41° \)
3. \( m \angle Y = 85° \)
4. \( m \angle F = 36° \)
5. \( \triangle ABC \) is similar to \( \triangle XYZ \) by AA similarity.
6. 4 ft
7. Both triangles contain the same angle at the far right, and a right angle, so the triangles are similar.
8. \( \frac{6}{9} = \frac{x}{33} \); \( x = 22 \)
9. \( m \angle RST = 79° \), \( m \angle VW = 33° \); congruent alternate interior angles were used to find the angle measures.
10. \( \triangle RST \) and \( \triangle WVT \) are similar by AA similarity since the triangles contain two congruent angles.

Reteach
1. | Lamp | Sign |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (ft)</td>
<td>( x )</td>
</tr>
<tr>
<td>Length of shadow (ft)</td>
<td>31.5</td>
</tr>
<tr>
<td>18 ft</td>
<td></td>
</tr>
</tbody>
</table>
2. | Woman | Son |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (ft)</td>
<td>5.5</td>
</tr>
<tr>
<td>Length of shadow (ft)</td>
<td>( 3 + 13.5 = 16.5 )</td>
</tr>
<tr>
<td>4.5 ft</td>
<td></td>
</tr>
</tbody>
</table>

Reading Strategies
1. the length of Zachary’s shadow
2. the height of the tree
3. the distance between the tree and Zachary
4. Both triangles contain angle $D$ and a right angle. The triangles are similar by AA similarity.

5. a. $ED$  
   b. $AB$

6. \[ \frac{x}{32} = \frac{4}{10}; \ 12.8 \text{ ft} \]

**Success for English Learners**

1. The angles are congruent, and the sides are proportional.
2. If two angles of one triangle are congruent to two angles of another triangle, the third angles are congruent and the triangles are similar.

**MODULE 11 Challenge**

1. \[ 180 = 58 + 9n − 8 + 7n + 2 \]
   \[ 180 = 52 + 16n \]
   \[ 128 = 16n \]
   \[ 8 = n \]
   \[ 9(8) − 8 = 72 − 8 = 64 \]
   \[ \angle x = 180 − 64 = 116 \]
   \[ m\angle x = 116^\circ \]

2. $m\angle x = 48^\circ$ Angle $x$ and the angle marked $48^\circ$ are alternate interior angles, and therefore congruent.

3. $m\angle x = 35^\circ + 45^\circ = 80^\circ$ Angle $x$ is made up of two alternate interior angles. Part is an alternate interior angle to a $45^\circ$ angle, and part is an alternate interior angle to a $35^\circ$ angle. I can use those measures to add because the angles are congruent.

4. $m\angle x = 70^\circ − 30^\circ = 40^\circ$ The angle marked $70^\circ$ and the angles marked $x$ and $30^\circ$ are alternate interior angles and therefore congruent. I can subtract $30^\circ$ from $70^\circ$ to find the missing part, $x$, of the angle.

**MODULE 12 The Pythagorean Theorem**

**LESSON 12-1**

Practice and Problem Solving: A/B

1. $c = 6.4$
2. $b = 20$
3. $a = 36$
4. $b = 18.2$
5. $a = 17.7$
6. $b = 72$
7. 10 blocks
8. a. Drawings will vary, but should show a rectangular solid 12 units high with a base 3 units wide and 4 units long
   b. 13 in.

Practice and Problem Solving: C

1. 1.4 in.
2. 3.5 km
3. 2.4 ft
4. 6.9
5. 16
6. 2.8
7. 17.3 m

Practice and Problem Solving: D

1. $c = 15$
2. $c = 26$
3. $c = 12.5$
4. 10.4 m
5. 134.2 yd
6. 6.7; 61; 7.8

Reteach

1. Drawings may vary, but should be squares of side 10. Sample:

2. $c = 17$ in.
3. $a = 10$ cm

**Reading Strategies**

1. side $D$; sides of length 6 and 12
2. the side connecting the ends of the 9 mm and 12-mm legs; sides of length 9 mm and 12 mm.
Success for English Learners
1. Legs: $\overline{AC}$ and $\overline{BC}$; hypotenuse: $\overline{AB}$
2. Step 2: 15; 25
   Step 3: 225; 625
   Step 4: 625; 225; 400
   Step 5: 400; 20

LESSON 12-2
Practice and Problem Solving: A/B
1. Yes; $7^2 + 24^2 = 25^2 = 625$
2. No; $30^2 + 40^2 = 2,500; 45^2 = 2,025$
3. Yes; $21.6^2 + 28.8^2 = 36^2 = 1,296$
4. No; $10^2 + 15^2 = 325; 18^2 = 324$
5. No; $10.5^2 + 36^2 = 1,406.25; 50^2 = 2,500$
6. Yes; $2.5^2 + 6^2 = 6.5^2 = 42.45$
7. No; $400^2 = 160,000; 200^2 + 300^2 = 130,000$
8. Width = 68.7 yd (approx.)
9. No; the third side would have to be 50, which is less than each of the other sides. Also, there are two “longest” sides, so neither could be a hypotenuse.

Practice and Problem Solving: C
1. not a right triangle; $15^2 = 225; 2(10)^2 = 200$
2. right triangle; $(2\sqrt{2})^2 = 8; 2(2)^2 = 8$
3. not a right triangle; $700^2 = 490,000; 2(300)^2 = 180,000$
4. No, because the hypotenuse has to be longer than either of the two legs.
5. No; $9^2 = 81; 4^2 + 8^2 = 80$
6. No; $(\sqrt{2})^2 + 1^2 = 2 + 1 = 3; 2x^2 + x^2$

8. Base: $(x\sqrt{2})^2 + x^2 = 5x^2$, so diagonal of base is $x\sqrt{5};$
   Solid diagonal: $(x\sqrt{5})^2 + x^2 = (\sqrt{6})^2$
   $6x^2 = 6; x = 1; $dimensions of the rectangular solid are 1 foot by 1 foot by 2 feet.

Practice and Problem Solving: D
1. 5
2. 13
3. $\sqrt{2}$
4. $\sqrt{13}$
5. no; $8^2 + 9^2 \neq 10^2$
6. no; $12^2 + 14^2 \neq 15^2$
7. yes; $10^2 + 24^2 = 26^2$
8. no; $14^2 + 15^2 \neq 21^2$
9. 120 yd; it’s the longest side.
10. 60 yd and 100 yd
11. 60; 100; 120
12. 3,600; 10,000; 14,400; 13,600; 14,400
13. No.
14. No; 120 yards would be the diagonal of the parking lot, and the two sides would be 60 yards and 100 yards, which would form a right triangle. The Pythagorean Theorem is not satisfied by the numbers 60, 100, and 120, so the triangle formed is not a right triangle.
Reteach
1. 10 in.
2. 15 mm
3. 3; $1^2 + 2^2 = 5; 3^2 = 9$; no
4. 8; $6^2 + 7^2 = 85; 8^2 = 64$; no
5. 25; $15^2 + 20^2 = 625; 25^2 = 625$; yes
6. $2^2 + 3^2 = 13; \left(\sqrt{13}\right)^2 = 13$
7. $3^2 + 6^2 = 45; \left(3\sqrt{5}\right)^2 = 9(5) = 45$

Reading Strategies
1. Answers may vary. Sample answer: “If a right triangle has sides of 5 and 12, then its third side is the square root of 5 squared plus 12 squared, or 13.”
2. Answers may vary. Sample answer: “If a triangle has sides 4, 4, and 8, then it is not a right triangle because the sum of 4 squared plus 4 squared is 32, which is not equal to 8 squared or 64.”

Success for English Learners
1. The sides 7, 24, and 25 can be used to make a right triangle as shown, assuming that the 7 and 24 sides are perpendicular to each other and form a right angle. However, this is an informal proof by observation, not a formal proof using specific numbers from the problem.
2. Shorter, since a length longer than 12 would make the square of the hypotenuse ($12^2 = 144$) greater than the sum of the squares of the sides of the sides ($5^2 + 8^2 = 25 + 64 = 89$).

LESSON 12-3

Practice and Problem Solving: A/B
1. $A(-4, 2); B(4, 6); C(4, 2)$
2. $D(-3, 3); E(3, -2); F(-3, -2)$
3. $\overrightarrow{AB}$
4. $\overrightarrow{DE}$
5. Answers will vary. Sample answer: 9 units.
6. Answers will vary. Sample answer: 8 units.

Practice and Problem Solving: C
1. $d_{AB} = \sqrt{10}; d_{BC} = 3; d_{AC} = 1; \overrightarrow{AB}$ is the hypotenuse, so does ($\sqrt{10}$)$^2 = 3^2 + 1^2$? Yes.
2. $AB = 3.5$ km; $BC = 2.5$ km; $CA = 2\sqrt{2}$, so the perimeter is $6 + 2\sqrt{2}$ or approx. 8.8 km.
3. $d = \sqrt{(x + 5)^2 + (3 - 7)^2} = \sqrt{(x + 5)^2 + 16}$; for $d = 5$, $(x + 5)^2 + 16 = 5$ and $x + 5 = 3$. So $x = -2$.
4. $d = \sqrt{(6 - 3)^2 + (y + 4)^2} = \sqrt{9 + (y + 4)^2}$; for $d = 5$, $\sqrt{9 + (y + 4)^2} = 5$ and $(y + 4)^2 = 16$ and $(y + 4) = -4$ and $y = -8$.

Practice and Problem Solving: D
1. $2\sqrt{2}$
2. $4\sqrt{2}; x_2 = -5; x_1 = -1; y_2 = 7; y_1 = 3$
   $d = \sqrt{(-5 + 1)^2 + (7 - 3)^2}$
3. $10\sqrt{2}; x_2 = 10; x_1 = 0; y_2 = -15; y_1 = -5$
   $d = \sqrt{(10 - 0)^2 + (-15 + 5)^2}$
4. Answers will vary.; Sample answer: $x$-distance between points = 10; $AB$ = more than 10.
5. Answers will vary.; Sample answer: $x$-distance between points = 5; $CD$ = more than 5.
6. Answers will vary.; Sample answer: $x$-distance between points = 5; $EF$ = more than 5.
7. Answers will vary.; Sample answer: $x$-distance between points = 7; $CD$ = more than 7.
8. The difference of the $y$-coordinates is $|5 - 1| = 4$.
9. The difference of the $y$-coordinates is $|-4 + 1| = 3$. 
10. The difference of the \( y \)-coordinates is \(|9 + 6| = 15\)

**Reteach**
1. horizontal
2. neither
3. vertical
4. neither
5. \( \sqrt{7.06} \)
6. \( \sqrt{17} \)

**Reading Strategies**
1. Yes
2. the Pythagorean Theorem
3. the Distance Formula
4. \( 6^2 + 8^2 = c^2 \)
   \( 36 + 64 = c^2 \)
   \( 100 = c^2 \)
   \( \sqrt{100} = c \)
   \( 10 = c \)
5. \( d = \sqrt{(10 - 2)^2 + (2 - 8)^2} \)
   \( d = \sqrt{8^2 + 6^2} \)
   \( d = \sqrt{64 + 36} \)
   \( d = \sqrt{100} \)
   \( d = 10 \)
6. Answers may vary, but should mention, at a minimum, that both of the last two steps involve finding a square root.

**Success for English Learners**
1. 6, 9, 4, 5; \( d = \sqrt{10} \) or about 3.2
2. 0, 1, 6, 8; \( d = \sqrt{5} \) or about 2.2

**MODULE 12 Challenge**
1. \( 3^2 + 4^2 = 5^2 \) because 9 + 16 = 25;
   \( a^2 + b^2 = c^2 \)
2. \( \frac{1}{2} (3^2) + \frac{1}{2} (4^2) = \frac{1}{2} (5^2) \) because
   \( 4.5 + 8 = 12.5 \)
3. \( \frac{\pi}{2} (3^2) + \frac{\pi}{2} (4^2) = \frac{\pi}{2} (5^2) \) because you can multiply both sides by \( \frac{2}{\pi} \);
   \( \frac{\pi}{2} (a^2) + \frac{\pi}{2} (b^2) = \frac{\pi}{2} (c^2) \)
4. \( \frac{1}{2} (6)(3) + \frac{1}{2} (8)(4) = \frac{1}{2} (10)(5) \) because
   \( 9 + 16 = 25; \)
   \( \frac{1}{2} (a) \bigg( \frac{a}{2} \bigg) + \frac{1}{2} (b) \bigg( \frac{b}{2} \bigg) = \frac{1}{2} (c) \bigg( \frac{c}{2} \bigg) \)
5. Answers will vary. Sample answer: using equilateral triangles:

**MODULE 13 Volume**

**LESSON 13-1**

**Practice and Problem Solving: A/B**
1. 2,122.6 cm\(^3\); 3.14 \( \cdot \) (6.5)\(^2\) \( \cdot \) 16 = 3.14 \( \cdot \) 42.25 \( \cdot \) 16 = 2,122.64 \( \approx \) 2,122.6
2. 37.7 cm\(^3\); 3.14 \( \cdot \) (2)\(^2\) \( \cdot \) 3 = 3.14 \( \cdot \) 4 \( \cdot \) 3 = 37.68 \( \approx \) 37.7
3. 9.4 ft\(^3\); 3.14 \( \cdot \) (1)\(^2\) \( \cdot \) 3 = 3.14 \( \cdot \) 1 \( \cdot \) 3 = 9.42 \( \approx \) 9.4
4. 70.7 in\(^3\); 3.14 \( \cdot \) (1.5)\(^2\) \( \cdot \) 10 = 3.14 \( \cdot \) 2.25 \( \cdot \) 10 = 70.65 \( \approx \) 70.7
5. 136 cm\(^3\); 3.14 \( \cdot \) (3.8)\(^2\) \( \cdot \) 3 = 3.14 \( \cdot \) 14.44 \( \cdot \) 3 = 136.0248 \( \approx \) 136
6. 413.8 cm\(^3\); 3.14 \( \cdot \) (3.3)\(^2\) \( \cdot \) 12.1 = 3.14 \( \cdot \) 10.89 \( \cdot \) 12.1 = 413.75466 \( \approx \) 413.8
7. a. 1,962.5 ft\(^3\); 3.14 \( \cdot \) (5)\(^2\) \( \cdot \) 25 = 3.14 \( \cdot \) 25 \( \cdot \) 25 = 1,962.5
   b. 5,298.8 ft\(^3\); 3.14 \( \cdot \) (7.5)\(^2\) \( \cdot \) 30 = 3.14 \( \cdot \) 56.25 \( \cdot \) 30 = 5,298.8
   c. 3,336.3 ft\(^3\); 5,298.8 – 1,962.5 = 3,336.3
Practice and Problem Solving: C
1. $8.8 \text{ ft}^3; 3.14 \cdot (0.75)^2 \cdot 5 = 8.83125$

2. a. 

<table>
<thead>
<tr>
<th>Candle Size</th>
<th>Radius</th>
<th>Height</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tall Candle</td>
<td>2 in.</td>
<td>10 in.</td>
<td>125.6 in.$^3$</td>
</tr>
<tr>
<td>Medium Candle</td>
<td>3 in.</td>
<td>6 in.</td>
<td>169.56 in.$^3$</td>
</tr>
<tr>
<td>Short Candle</td>
<td>2 in.</td>
<td>4 in.</td>
<td>200.96 in.$^3$</td>
</tr>
</tbody>
</table>

b. The short candle has the most wax.

c. $75.36 \text{ in}^3; 200.96 - 125.6 = 75.36$

d. $125.6 < 169.56 < 200.96$

3. $628 = (3.14)(r^2)(8)$

   $628 = 25.12r^2$

   $25 = r^2$

   $5 = r$

Reading Strategies
1. the formula for the area of a circle ($A = \pi r^2$ and $B = \pi r^2$)

2. because you must multiply the area of the base (expressed in in.$^2$), by the height (expressed in in.); so, in.$^2$ multiplied by in. results in the volume being in in.$^3$

3. Answers will vary. Sample answer: A cylinder has a radius of 5 inches and a height of 12 inches. What is the volume of the cylinder? Round the answer to the nearest tenth. (942 in.$^3$)

Success for English Learners
1. the formula for the area of a circle ($A = \pi r^2$ and $B = \pi r^2$)

2. because you must multiply the area of the base (expressed in in.$^2$), by the height (expressed in in.); so, in.$^2$ multiplied by in. results in the volume being in in.$^3$

3. Answers will vary. Sample answer: A cylinder has a radius of 5 inches and a height of 12 inches. What is the volume of the cylinder? Round the answer to the nearest tenth. (942 in.$^3$)

LESSON 13-2

Practice and Problem Solving: A/B
1. $6,358.5 \text{ in.}^3; \frac{1}{3} \cdot 3.14 \cdot (15)^2 \cdot 27 = \frac{1}{3} \cdot 3.14 \cdot 225 \cdot 27 = \frac{1}{3} \cdot 19,075.5 = 6,358.5$

2. $3,299.2 \text{ m}^3; \frac{1}{3} \cdot 3.14 \cdot (12.4)^2 \cdot 20.5 = \frac{1}{3} \cdot 3.14 \cdot 153.76 \cdot 20.5 = \frac{1}{3} \cdot 9,897.53 = 3,299.18 \approx 3,299.2$
3. \(25.1 \text{ in.}^3; \frac{1}{3} \cdot 3.14 \cdot (2)^2 \cdot 6 = \frac{1}{3} \cdot 3.14 \cdot 4 \cdot 6 = \frac{1}{3} \cdot 75.36 = 25.12 \approx 25.1\)

4. \(167.5 \text{ cm}^3; 3.14 \cdot (4)^2 \cdot 10 = \frac{1}{3} \cdot 3.14 \cdot 16 \cdot 10 = 167.5\)

5. \(339.1 \text{ in.}^3; \frac{1}{3} \cdot 3.14 \cdot (4.5)^2 \cdot 16 = \frac{1}{3} \cdot 3.14 \cdot 20.25 \cdot 16 = \frac{1}{3} \cdot 1,017.36 = 339.12\)

6. \(392.5 \text{ cm}^3; \frac{1}{3} \cdot 3.14 \cdot (5)^2 \cdot 15 = \frac{1}{3} \cdot 3.14 \cdot 25 \cdot 15 = \frac{1}{3} \cdot 1,177.5 = 392.5\)

7. a. \(1,236.375 \text{ ft}^3; \frac{1}{3} \cdot 3.14 \cdot (15)^2 \cdot 21 = 4,945.5 \text{ ft}^3\)

b. \(4,592.25 \text{ ft}^3; 3.14 \cdot (15)^2 \cdot 26 = 18,369 \text{ ft}^3\)

c. \(23,314.5 \text{ ft}^3\)

Practice and Problem Solving: C

1. a.

<table>
<thead>
<tr>
<th>Cone Size</th>
<th>Radius</th>
<th>Height</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cone A</td>
<td>2 cm</td>
<td>10 cm</td>
<td>41.87 cm$^3$</td>
</tr>
<tr>
<td>Cone B</td>
<td>2 cm</td>
<td>20 cm</td>
<td>83.73 cm$^3$</td>
</tr>
<tr>
<td>Cone C</td>
<td>4 cm</td>
<td>10 cm</td>
<td>167.47 cm$^3$</td>
</tr>
</tbody>
</table>

b. Cone C has the greatest volume.

c. \(41.9 \text{ cm}^3; 83.73 - 41.86 = 41.87 \approx 41.9\)

d. \(41.9 < 83.7 < 167.5\)

e. when you double the radius because the radius is squared

2. \(732.7 = \frac{1}{3} \cdot (3.14)(r^2)(28)\)

\[
732.7 = 29.3r^2 \\
25 = r^2 \\
5 = r \\
d = 2r = 2(5) = 10 \text{ in.}
\]

Practice and Problem Solving: D

1. \(V = \frac{1}{3} \pi r^2 h\)

\(V = \frac{1}{3} \cdot 3.14 \cdot 3^2 \cdot 5\)

\(V = \frac{1}{3} \cdot 3.14 \cdot 9 \cdot 5\)

\(V = 47.1 \text{ cm}^3\)

2. \(V = \frac{1}{3} \pi r^2 h\)

\(V = \frac{1}{3} \cdot 3.14 \cdot 5^2 \cdot 9\)

\(V = \frac{1}{3} \cdot 3.14 \cdot 25 \cdot 9\)

\(V = 235.5 \text{ ft}^3\)

3. \(100.5 \text{ in.}^3\)

4. \(130.8 \text{ cm}^3\)

5. \(46.1 \text{ cm}^3\)

6. \(20 \text{ in.}^3\)

Reteach

1. radius \(r\) of base = 3 in.

\(V = \frac{1}{3} Bh\)

\(V = \frac{1}{3} \cdot 3.14 \cdot 3 \cdot 10\)

\(V = \frac{1}{3} \cdot (\pi \times 3^2) \times 10\)

\(V = \frac{1}{3} \cdot (28.26) \times 10\)

\(V = 9.42 \times 10\)

\(V = 94.2 \text{ in.}^3\)

2. radius \(r\) of base = 6 cm

\(V = \frac{1}{3} Bh\)

\(V = \frac{1}{3} \cdot 3.14 \cdot 6 \cdot 10\)

\(V = \frac{1}{3} \cdot (\pi \times 6^2) \times 4\)

\(V = \frac{1}{3} \cdot (113.04) \times 4\)

\(V = 37.68 \times 4\)

\(V = 150.72 \text{ cm}^3\)
Reading Strategies

1. the diameter
2. 5 cm
3. 78.5 cm$^2$ $B = \pi r^2 = 3.14 \cdot 5^2 = 3.14 \cdot 25 = 78.5$ cm$^3$
4. 10 cm
5. $V = \frac{1}{3} Bh$
   
   $V = \frac{1}{3} \cdot 785$
   
   $V = 261.666 \approx 261.7$ cm$^3$

Success for English Learners

1. To find the radius, you must divide the diameter by 2.

2. Answers will vary. Sample answer: A paperweight is in the shape of a cone with a diameter of 3 in. and a height of 4 in. What is the volume of the cone? $V = \frac{1}{3}$
   
   $Bh = \frac{1}{3} \cdot 3.14 \cdot 1.5^2 \cdot 4 = \frac{1}{3} \cdot 3.14 \cdot 2.25 \cdot 4 = \frac{1}{3} \cdot 28.26 = 9.42$ in.$^2$

LESSON 13-3

Practice and Problem Solving: A/B

1. $\frac{4}{3} (3.14)(5)^3 = \frac{4}{3} (3.14)(125) \approx 523.3333 \approx 523.3$ in.$^3$
2. $\frac{4}{3} (3.14)(1.2)^3 = \frac{4}{3} (3.14)(1.728) = 7.23456 \approx 7.2$ m$^3$

3. $\frac{4}{3} (3.14)(3)^3 = \frac{4}{3} (3.14)(27) = 113.04 \approx 113$ in.$^3$
4. $\frac{4}{3} (3.14)(4.5)^3 = \frac{4}{3} (3.14)(91.125) = 381.51 \approx 381.5$ m$^3$
5. $\frac{4}{3} (3.14)(1.5)^3 = \frac{4}{3} (3.14)(3.375) = 14.13 \approx 14.1$ m$^3$
6. $\frac{4}{3} (3.14)(8)^3 = \frac{4}{3} (3.14)(512) \approx 2,143.5733 \approx 2,143.6$ in.$^3$

7. $\frac{4}{3} (3.14)(4.3)^3 \approx \frac{4}{3} (3.14)(79.507) = 332.8693 \approx 332.9$ in.$^3$

8. a. $\frac{4}{3} (3.14)(1.25)^3 \approx \frac{4}{3} (3.14)(1.953125) \approx 8.177 \approx 8.2$ in.$^3$
   
   b. $\frac{4}{3} (3.14)(1.3125)^3 = \frac{4}{3} (3.14)(2.261) \approx 9.4661 \approx 9.5$ in.$^3$
   
   c. $8.2$ in.$^3 < x < 9.5$ in.$^3$

Practice and Problem Solving: C

<table>
<thead>
<tr>
<th>Cone Size</th>
<th>Radius</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Sphere</td>
<td>3 in.</td>
<td>113 in.$^3$</td>
</tr>
<tr>
<td>Mini Sphere</td>
<td>1.5 in.</td>
<td>14.1 in.$^3$</td>
</tr>
<tr>
<td>Maxi Sphere</td>
<td>6 in.</td>
<td>904.3 in.$^3$</td>
</tr>
</tbody>
</table>

b. The mini sphere is $\frac{1}{8}$ the volume of the basic sphere.

c. The volume of the maxi sphere is 8 times greater than the volume of the basic sphere.
d. $14.1 < 113 < 904.3$

2. $V = \frac{4}{3} \pi r^3$.

$4,186 = \frac{4(3.14)}{3} r^3 = \frac{12.56 r^3}{3}$

$12,558 = 12.56 r^3$

$r^3 \approx 999.84$

$r \approx 9.999 \approx 10$ in.
3. No. The volume of the sphere is always about 4.1866… times the volume of the cube when the radius of the sphere and the side of the cube are the same measure. The volume of the cube is the side raised to the power of 3, whereas the volume of the sphere is the same number raised to the power of 3 which is then multiplied by \( \pi \) and by 4 and finally divided by 3.

Practice and Problem Solving: D

1. \( V = \frac{4}{3} \pi r^3 \)
   \[ V = \frac{4}{3} \times 3.14 \times 9^3 \]
   \[ V = \frac{4}{3} \times 3.14 \times 729 \]
   \[ V \approx 3,052.1 \text{ cm}^3 \]

2. \( V = \frac{4}{3} \pi r^3 \)
   \[ V = \frac{4}{3} \times 3.14 \times 2^3 \]
   \[ V = \frac{4}{3} \times 3.14 \times 8 \]
   \[ V \approx 33.5 \text{ m}^3 \]

3. \( V = \frac{4}{3} \pi r^3 \)
   \[ V = \frac{4}{3} \times 3.14 \times 2^3 \]
   \[ V = \frac{4}{3} \times 3.14 \times 8 \]
   \[ V \approx 33.5 \text{ cm}^3 \]

4. \( V = \frac{4}{3} \pi r^3 \)
   \[ V = \frac{4}{3} \times 3.14 \times 5^3 \]
   \[ V = \frac{4}{3} \times 3.14 \times 125 \]
   \[ V \approx 523.3 \text{ m}^3 \]

5. \( V = \frac{4}{3} \pi r^3 \)
   \[ V = \frac{4}{3} \times 3.14 \times 2.8^3 \]
   \[ V = \frac{4}{3} \times 3.14 \times 22 \]
   \[ V \approx 91.9 \text{ cm}^3 \]

6. \( V = \frac{4}{3} \pi r^3 \)
   \[ V = \frac{4}{3} \times 3.14 \times 1^3 \]
   \[ V = \frac{4}{3} \times 3.14 \times 1 \]
   \[ V \approx 4.2 \text{ m}^3 \]

Reteach

1. diameter = 2.5 in.
   radius = 1.25 in.
   \[ V = \frac{4}{3} \times 3.14 \times 1.25^3 \]
   \[ V = \frac{4}{3} \times 3.14 \times 1.95 \]
   \[ V = 8.1771 \]
   \[ V \approx 8.2 \text{ in.}^3 \]

2. diameter = 12 cm
   radius = 6 cm
   \[ V = \frac{4}{3} (3.14 \times 6^3) \]
   \[ V = \frac{4}{3} (3.14 \times 216) \]
   \[ V = 904.32 \]
   \[ V \approx 904.3 \text{ cm}^3 \]

Reading Strategies

1. Answers will vary. Sample answers: baseball, basketball, marble, orange
2. diameter
3. radius
4. Divide the diameter by 2.
5. Multiply the radius by 2.
Success for English Learners

1. The expression cm$^3$ in the answer is the unit of measurement, and it stands for cubic centimeters.

2. Answers will vary. Sample answer: What is the volume of a sphere with a radius of 6 inches?

\[ V = \frac{4}{3} \times 3.14 \times 6^3 = 904.32 \text{ in.}^3 \]

MODULE 13 Challenge

1. Check students’ drawings for correct labeling and proportionality to given measures.

Sample answer. Other answers are possible.

\[ V = \pi r^2 h \]
\[ V \approx (3.14)(4.5^2)(7) \]
\[ V \approx (3.14)(20.25)(7) \]
\[ V \approx 445.095 \text{ in.}^3 \]
\[ V \approx 445.1 \text{ in.}^3 \]

2. Check students’ drawings for correct labeling and proportionality to given measures.

Sample answer. Other answers are possible.

\[ V = \frac{1}{3} \pi r^2 h \]
\[ V \approx \frac{1}{3}(3.14)(5^2)(3.5) \]
\[ V \approx \frac{1}{3}(3.14)(25)(3.5) \]
\[ V \approx 91.58 \text{ in.}^3 \]

3. Check students’ drawings for correct labeling and proportionality to given measures.

Sample answer. Other answers are possible.

\[ V = \frac{4}{3} \pi r^3 \]
\[ V \approx \frac{4}{3}(3.14)(6.2^3) \]
\[ V \approx \frac{4}{3}(3.14)(238.33) \]
\[ V \approx 997.81 \text{ cm}^3 \]