AP Calculus (BC)

This packet is a review of some Precalculus topics and some Calculus topics. It is to be done NEATLY and on a SEPARATE sheet of paper. Use your discretion as to whether you should use a calculator or not. When in doubt, think about if I would use one – that should guide you! Points will be awarded only if the correct work is shown, and that work leads to the correct answer. Have a great summer and I am looking forward to seeing you in September. ☺

Part I: PreCalculus!

1) For what value of *k* are the two lines $2x + ky = 3$ and $x + y = 1$

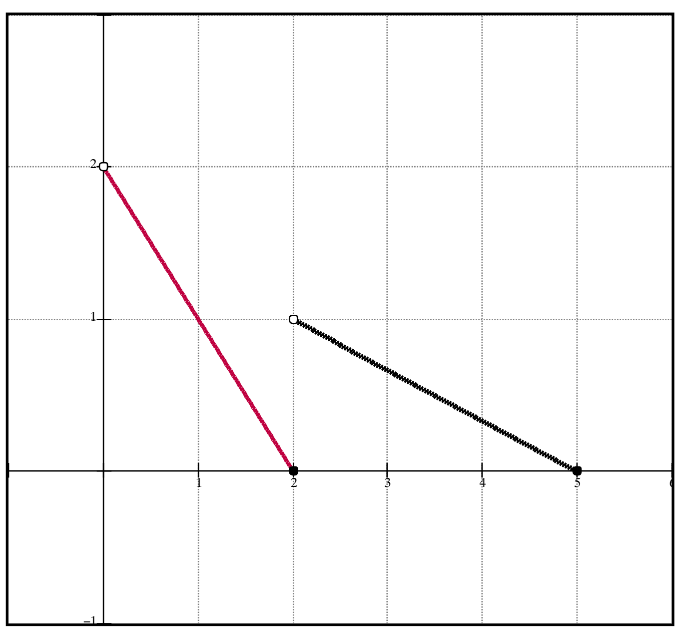
(a) parallel? (b) perpendicular?

2) Consider the circle of radius 5 centered at (0, 0). Find an equation of the line tangent to the circle at the point (3, 4) in slope intercept form.

3) Graph the function shown below. Also indicate any key points and state the domain and range.

$$f(x) = \begin{cases} 4 - x^2, & x < 1 \\ \frac{3}{2}x + \frac{3}{2}, & 1 \le x \le 3 \\ x + 3, & x > 3 \end{cases}$$

4) Write a piecewise formula for the function shown. Include the domain of each piece!



5) Graph the function $y = 3e^{-x} - 2$ and indicate asymptote(s). State its domain, range, and intercepts.

For #6-7, parametric equations are given. Complete the table and sketch the curve represented by the parametric equations (label the initial and terminal points as well as indicate the direction of the curve).

6) $x = 4\sin t, \quad y = 2\cos t, \quad 0 \le t \le 2\pi$

t	***0***	$\pi/4$	$\pi/2$	$3\pi/4$	π	$3\pi/2$	2π
x							
y							

7) $x = 2t - 5, \quad y = 4t - 7, \quad -2 \le t \le 3$

t	***– 2***	***– 1***	***0***	***1***	***2***	***3***
x						
y						

Part II: Continuity

For #1-4 below, find the limits, if they exist.(#1-13 are 1 pt each)

1) $\lim_{x \to 4} \frac{2x^3 - 7x^2 - 4x}{x - 4}$

2) $\lim_{x \to 9} \frac{\sqrt{x} - 3}{9 - x}$

3) $\lim_{x \to 1} \frac{x^2 - 2x - 5}{x + 1}$

4) $\lim_{x \to -2} \frac{x^3 + 8}{x + 2}$

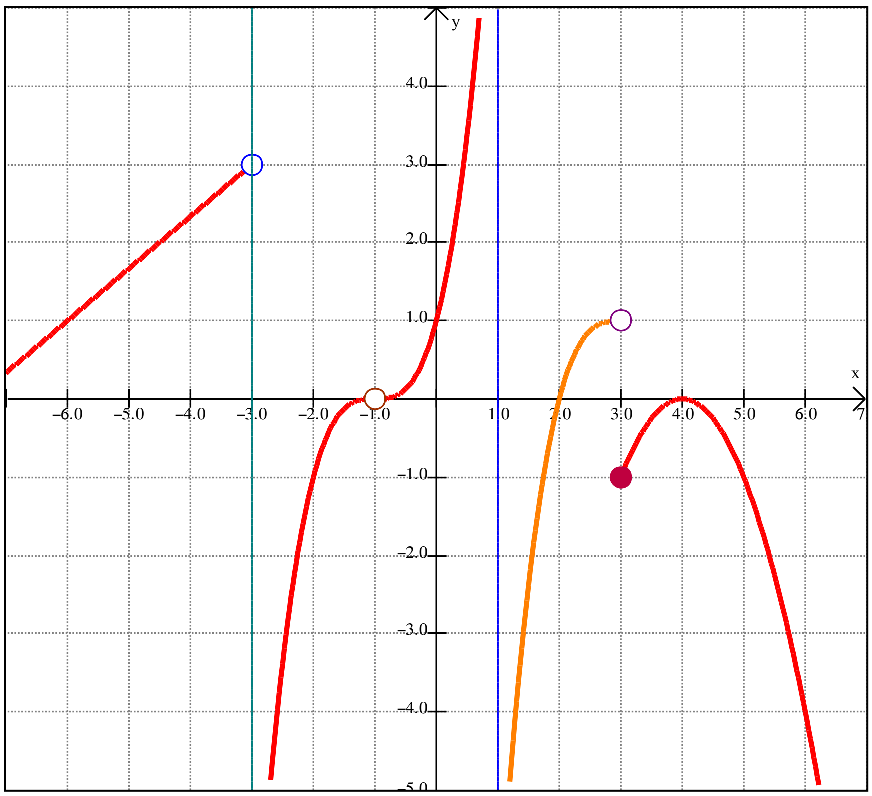
For #5-7, explain why each function is discontinuous and determine if the discontinuity is removable or nonremovable.

5) $g(x) = \begin{cases} 2x - 3, & x < 3 \\ -x + 5, & x \ge 3 \end{cases}$

6) $b(x) = \frac{x(3x + 1)}{3x^2 - 5x - 2}$

7) $h(x) = \frac{\sqrt{x^2 - 10x + 25}}{x - 5}$

For #8-13, determine if the following limits exist, based on the graph below of p(x). If the limits exist, state their value. Note that x = -3 and x = 1 are vertical asymptotes.



8) $\lim_{x \to 1^-} p(x)$

9) $\lim_{x \to -3^-} p(x)$

10) $\lim_{x \to 2} p(x)$

11) $\lim_{x \to 3^-} p(x)$

12) $\lim_{x \to 3^+} p(x)$

13) $\lim_{x \to -1} p(x)$

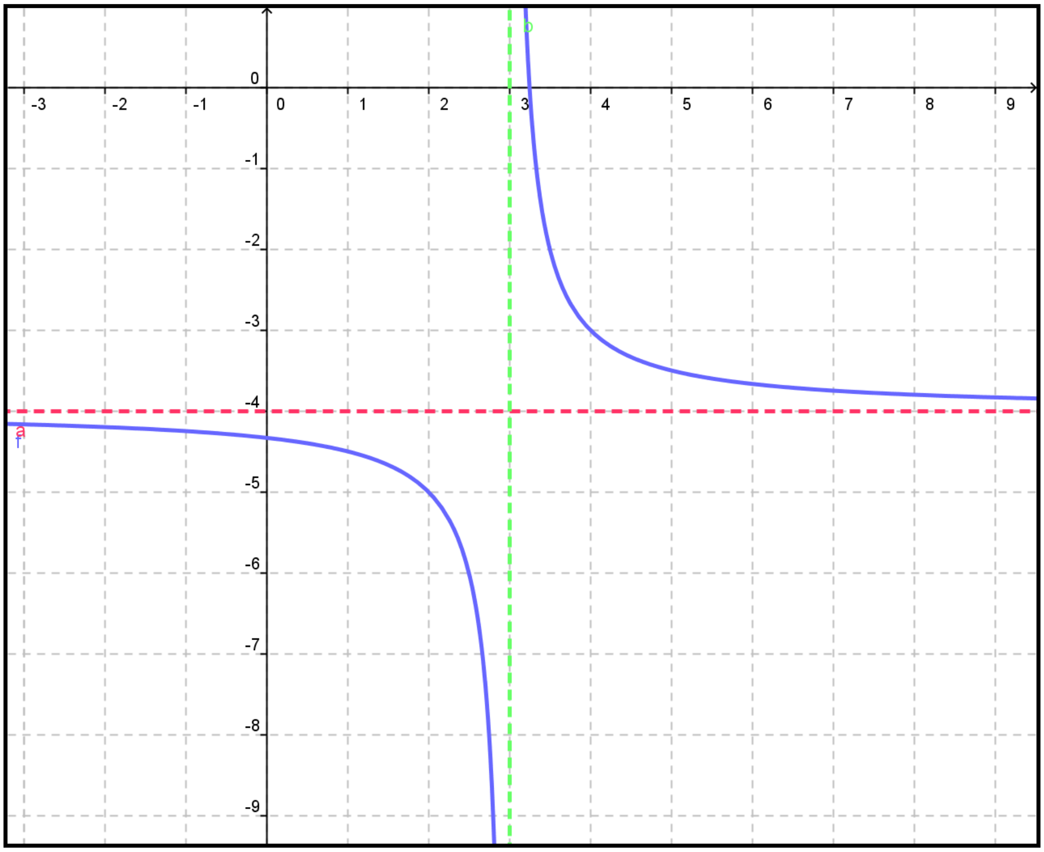
14) Consider the function $f(x) = \begin{cases} x^2 + kx & x \le 5 \\ 5\sin\left(\frac{\pi}{2}x\right) & x > 5 \end{cases}$,

In order for the function to be continuous at x = 5, the value of *k* must be

15) Consider the function $f(x) = \begin{cases} \frac{\sin x}{x} & x \ne 0 \\ k & x = 0 \end{cases}$.

In order for the function to be continuous at x = 0, the value of *k* must be

Use the graph of f(x), shown below, to answer #16-18. (1 pt each).



16) For what value of a is $\lim_{x \to a} f(x)$ nonexistent?

17) $\lim_{x \to \infty} f(x) =$

18) $\lim_{x \to -\infty} f(x) =$

Part III: The Derivative

1) $$\lim_{h \to 0} \frac{\tan^{-1}(1+h) - \frac{\pi}{4}}{h} =$$

2) $$\lim_{h \to 0} \frac{\sec(\pi + h) - \sec(\pi)}{h} =$$

For #3-8, find the derivative.

3) $y = \ln(1+e^x)$

4) $y = \csc(1+\sqrt{x})$

5) $y = (\tan^2 x)(3\pi x - e^{2x})$

6) $y = \sqrt[7]{x^3 - 4x^2}$

7) $f(x) = (x+1)e^{3x}$

8) $f(x) = \dfrac{e^{x/2}}{\sqrt{x}}$

9) Consider the function $f(x) = \sqrt{x-2}$. On what intervals are the hypotheses of the Mean Value Theorem satisfied?

10) If $xy^2 - y^3 = x^2 - 5$, then $\dfrac{dy}{dx} =$

11) The distance of a particle from its initial position is given by $s(t) = t - 5 + \dfrac{9}{(t+1)}$, where *s* is feet and *t* is minutes. Find the velocity at *t* = 1 minute in appropriate units.

Use the table below for #12-13.

X	$f(x)$	$g(x)$	$f'(x)$	$g('x)$
1	4	2	5	½
3	7	-4	$\frac{3}{2}$	-1

12) The value of $\dfrac{d}{dx}(f \cdot g)$ at x = 3 is

13) The value of $\dfrac{d}{dx}\left(\dfrac{f}{g}\right)$ at x = 1 is

In #14-15, use the table below to find the value of the first derivative of the given functions for the given value of x.

X	$f(x)$	$g(x)$	$f'(x)$	$g('x)$
1	3	2	0	¾
2	7	-4	$\frac{1}{3}$	-1

14) $[f(x)]^2$ at x = 2 is

15) $f(g(x))$ at x = 1 is

16) Let f be the function defined by $f(x) = \frac{x + \sin x}{\cos x}$ $for\ -\frac{\pi}{2} < x < \frac{\pi}{2}$.

(a) State whether *f* is an even function or an odd function. Justify your answer..
(b) Find $f'(x)$.
(c) Write an equation for the line tangent to the graph of *f* at the point $(0, f(0))$.

Part IV: More Derivatives and Applications

For #1-3, find all critical values, intervals of increasing and decreasing, any local extrema, points of inflection, and all intervals where the graph is concave up and concave down.

1) $f(x) = \frac{5 - 4x + 4x^2 - x^3}{x - 2}$

2) $y = 3x^3 - 2x^2 + 6x - 2$

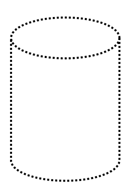
3) $f'(x) = 5x^3 - 15x + 7$

4) The graph of the function $y = x^5 - x^2 + \sin x$ changes concavity at x =

5) Find the equation of the line tangent to the function $y = \sqrt[4]{x^7}$ at $x = 16$.

6) For what value of x is the slope of the tangent line to $y = x^7 + \frac{3}{x}$ undefined?

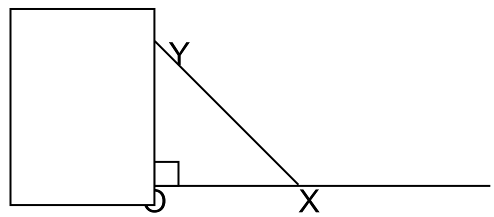
7)



The balloon shown is in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of 261π cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters, the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute. (The volume of a cylinder with radius *r* and height *h* is $\pi r^2 h$, and the volume of a sphere with radius *r* is $\frac{4}{3}\pi r^3$.)

(a) At this instant, what is the height of the cylinder?
(b) At this instant, how fast is the height of the cylinder increasing?

8)



A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at a constant rate of ½ foot per second.

(a) Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.

(b) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building.

10) *Find* $\frac{dy}{dx}$ $\quad x = \sqrt[9]{t}$, $\quad y = 5 - t$

11) *Find the second derivative* $\frac{d^2y}{dx^2}$ $\quad x = 4t^2, y = \ln t$

12) Find two sets of polar coordinates for the point for $0 \le \theta \le 2\pi$. $\quad \left(7\sqrt{3}, 7\right)$

13) *Find* $\frac{dy}{dx}$ $\quad r = 6\cos(3\theta)$